This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

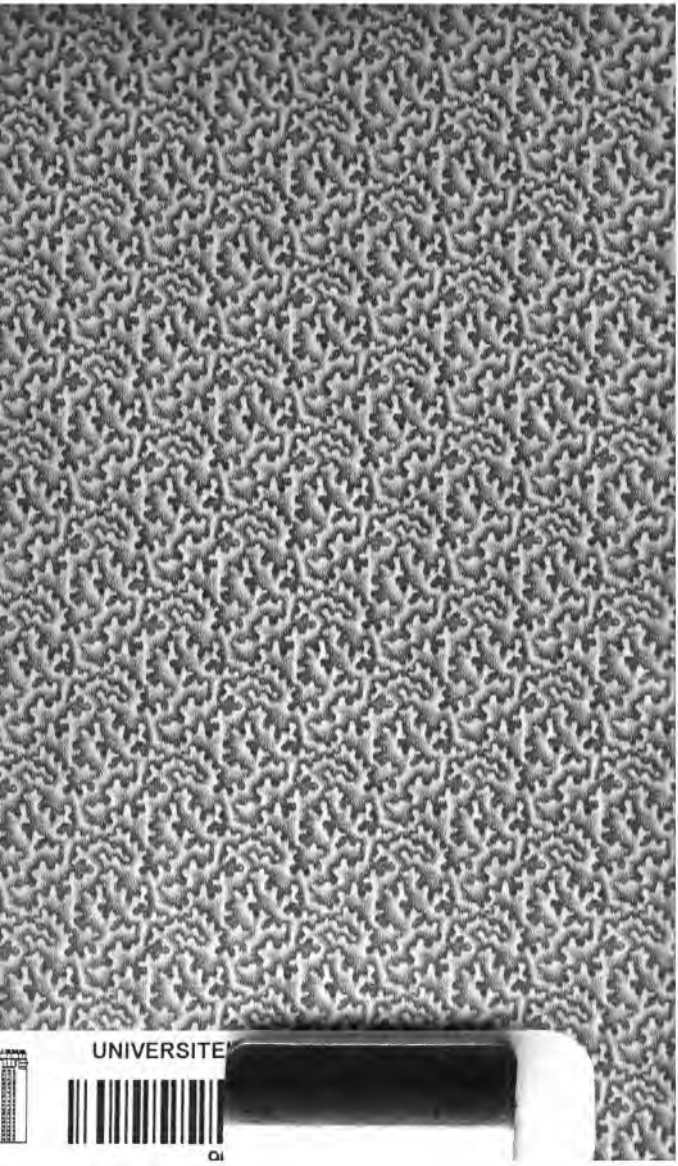
+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

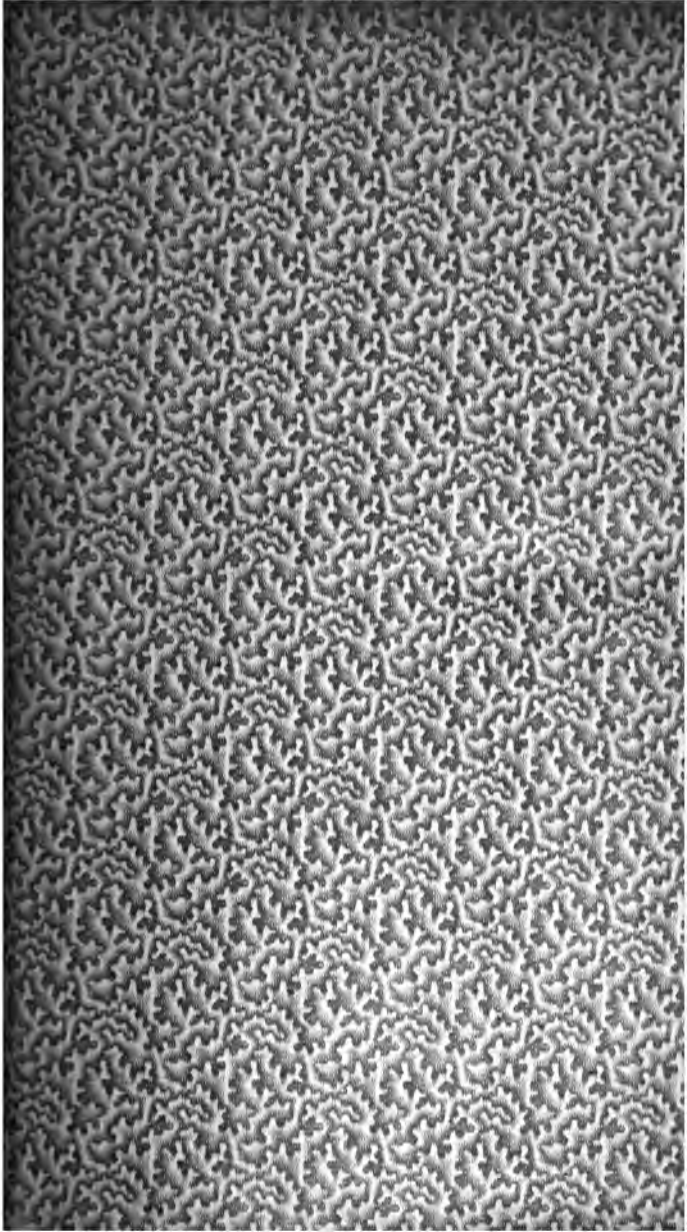
+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`


UNIVERSITE



Ma. 973

Ma. 973

Mat 973

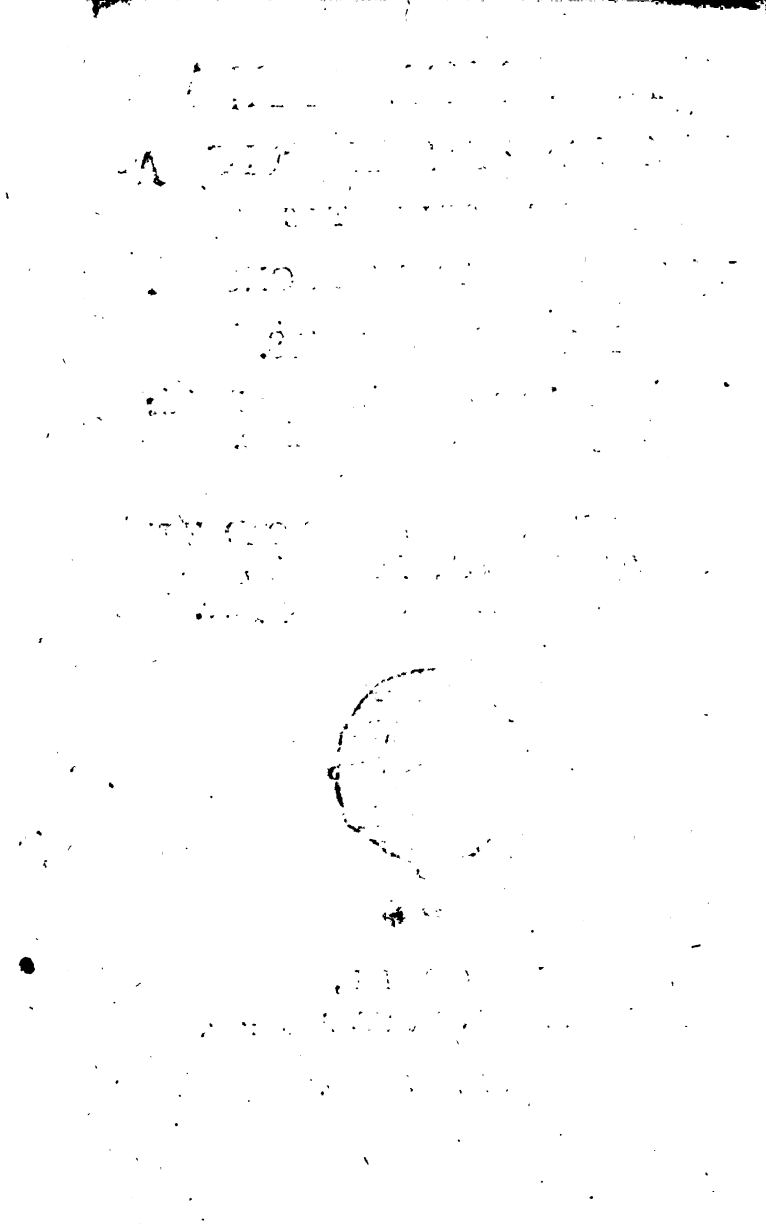
TRIGONOMETRIA
PLANA ET SPHÆRICA
CUM SELECTIS
EX GEOMETRIA ET ASTRONOMIA
PROBLEMATIS.

Accedunt Sinuum Canones, & ex Euclide Propositiones magis necessariæ.

AUCTORE
R. P. JACOBO GOODEN
Societatis JESU, in Collegio Anglorum Leodii Matheseos Professore.

LEODII,
Apud J. F. BRONCART. Anno 1704.

Superiorum Permissu.



PRÆFATIO.

AD Tractatum hunc de Trigonometria conscribendum impulêre tum ipsius scientiæ summa necessitas & præstantia, tum multò magis infrequentia scriptorum, qui in explicandis Matheseos rudimentis se Tyronum captui submittant. Plerique enim vel brevitatis plus æquo studiosi, Lectores suos non tam efficiunt peritos, quàm præsupponunt; vel perplexâ rerum varietate impares discentium animos obruunt & confundunt: ita ut difficile sit quempiam invenire qui omnia Trigonometriæ præcepta clarè succinctèque demonstret. Hinc fit ut plerique Matheseos Candidati hæreant in ipso limine, vel librorum subsidio destituti, vel nimiâ eorundem mole deterriti.

Ut huic incommodo occurreretur, Opus hoc incipientium ingeniis ita attemperare conatus sum, ut neque moles fastidium pareret, nec brevitas obscuritatem. Selegi proinde ea, quorum notitiam longa multorum experientia docuit necessariam; quæ ad finem propositum minùs sunt idonea, utcunque scitu digna, penitùs relegavi; quæ verò talia sunt ut juvare quidem possint, non tamen prorsus requirantur, vel obiter perstrinxi, vel curavi ita à reliquis secernenda, ut neque officere possint incipienti, neque à proficiente desiderari.

Trigonometriæ Planæ subjunxi problemata selecta, quibus præcipuam Practicæ Geometriæ partem sum complexus; tum ut essent exemplorum loco, tum ut Tyrones præsenti laborum fructu recreatos ad magis

ardua confirmarent; idem quoque post Sphæricam absolutam eodem consilio præstiti, problematis illustrioribus ex Astronomia non sine delectu excerptis, iis præsertim quæ in Horographia, aliisque Matheseos partibus usui sunt futura.

Denique ut ad manum haberentur omnia, quæ scientiæ hujus Theoria vel Praxis exigit, selegi ex Euclide elementa maximè necessaria; quibus adjeci Sinuum & Tangentium Canones, cum numerorum Logarithmis, quales ad Geometricas operationes magis obvias requiruntur & sufficiunt. Quæ omnia conatus sum in talem ordinem redigere, ut sperem à quovis posse facilè percipi, & inoffenso pede percurri. Methodum autem quam in præcipuis demonstrationibus secutus sum, antiquâ longè commodiorem esse facilè intelliget, qui horum brevitatem cum aliorum prolixitate contulerit.

Notanda.

1. Elementa eo ordine citantur, quo habentur in selectis ad libri hujus calcem appositis.

2. In propositionibus citandis *p* significat ipsam propositionem, *c* caput, *h* hujus, *P* partem. Reliqua verborum compendia quivis per se intelliget.

3. Proportiones brevitatis causâ sic exprimo. A : B :: C : D. Lege, A est ad B, ut C ad D

4. Quando sermo est de Figura aliqua, quæ in ipso loco non apponitur; respicienda est Figura proximè præcedens, nisi aliud indicetur.

TRACTATUS DE TRIGONOMETRIA.

Rianguli Resolutio, quæ Græco nomine *Trigonometria* dicitur, non aliud est quàm alicujus in triangulo propositi lateris aut anguli inventio: & quia triangula alia sunt plana seu rectilinea, alia sphærica; hinc fit ut Trigonometria, pro objectorum diversitate, in *Planam* & *Sphæricam* dividatur.

PARS PRIMA.

DE TRIGONOMETRIA PLANA.

PArtem hanc, quæ de Trigonometriâ planâ tota est; in sex capita dividimus; quatuor priora totam de triangulis rectilineis doctrinam complectentur: in reliquis insignem Trigonometriæ in quibusvis dimensionibus investigandis usum indicabimus.

CAPUT I.

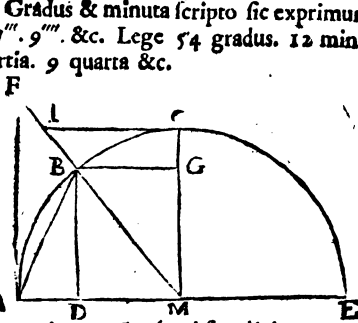
Præmittuntur aliqua ad Trigonometriam Planam.

DEFINITIONES.

1. CIrculi circumferentiam dividunt Mathematici in partes 360. quas vocant *Gradus*; unde semicircumferentia gradus continet 180. quadrans 90.

2. *Minutum* est 60a pars Gradûs: *Secundum* 60a pars minuti: *Tertium* 60a pars secundi, & sic deinceps.

Gradus & minuta ſcripto ſic exprimuntur: 54 gr. 12'. 34''. 17'''. 9''''. &c. Lege 54 gradus. 12 minuta. 34 ſecunda. 17 tertia. 9 quarta &c.



3. Angulus aliquis (AMB) notus dicitur, quando cognoſcuntur gradus arcûs (AB) ſubtenſi, qui ex (M) anguli vertice, tanquam centro, deſcribitur.

4. *Sinus rectus* (qui ſimpliciter vocatur ſinus) anguli alicujus (AMB, vel BME) eſt recta (BD) ducta ab arcûs ſubtenſi (AB, vel BE) uno extremo (B) perpendiculariter ad diametrum (AE) tranſeuntem per ejuſdem arcûs alterum extremum (A vel E)

5. *Sinus verſus* alicujus anguli (AMB) eſt diametri pars (AD) intercepta inter ſinum rectum (BD) & arcum (AB) angulo ſubtenſum.

6. Sinus anguli recti (AMC) ſeu *Sinus totus* eſt ipſe circuli radius (CM) ut patet ex defin. 4.

7. *Tangens* alicujus anguli (AMB) eſt recta (AF) ab arcûs ſubtenſi (AB) uno extremo (A) erecta perpendiculariter ad diametrum (AE) & producta donec occurrat rectæ (MF) ductæ à centro (M) per (B) alterum extremum ejuſdem arcûs (AB)

8. Recta (MF) cui tangens ſic occurrit, eſt *ſecans* ejuſdem anguli (AMB)

Nota eoſdem ſemper eſſe angulorum & arcuum ſubtenſorum ſinus, tangentes, & ſecantes.

9. *Complementum* arcûs vel anguli ad quadrantem vel angulum rectum, eſt ejuſdem arcûs vel anguli differentia (ſive per exceſſum, ſive per defectum) à quadrante vel angulo recto. Hinc angulorum (AMC, BME) complementum eſt angulus (BMC) qui uni (AMC) additus, alteri (BME) ſubtractus relinquit angulum rectum (CMA, vel CME.) Similiter arcuum (AB, BE) complementum eſt arcus (CB) eorum differentia à quadrante (CA vel CE.)

Complementi CB, vel CMB sinus Rectus est BG, sinus Versus CG, Tangens CI, Secans MI.

10. Complementum arcûs vel anguli ad semicirculum vel duos rectos, est arcus vel angulus qui reliquus est ad semicirculum vel duos rectos.

In Trigonometria Plana vel citantur, vel præsupponuntur sequentes Euclidis Propositiones; quas proinde diligenter legerit oportet, ac intellexerit quisquis ad hanc Matheseos partem accedit.

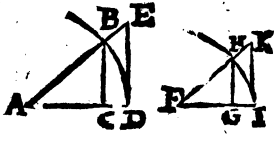
Ex Libro 1°. Definitiones & Axiomata. Prop. 1. 4. cum scholio. 5 6. 8. 9. 10. 12. 13. 15. 22. 23. cum scholio. 27. 29. 32. cum 1. 4. & 5. Corol. 34. 35. 37. 38. 41. 47.

Ex Libro 3°. Definitiones omnes, & Prop. 3. 20. 21. 22.

Ex Libro 5°. Definitiones omnes, & Prop. 7. 12. 15. 16 cum 2 Corol. 17. 18.

Ex Libro 6°. Definitiones omnes, & Prop. 1. 2. 4 cum 1 & 2 Corol. 5. 6. 33.

PROPOSITIO I.



In circulis quibuscunque eadem est ratio radiorum (AB, FH) *ad sinus* (BC, HG) *tangentes*, (ED, KI) *& secantes* (AE, FK) *similium arcuum* (BD, HI) *& angulorum æqualium* (A & F)

Demonstr. In triangulis ABC, FGH anguli A & F sunt (*ex suppos.*) æquales, & anguli C & G (*per 4 def. 6*) recti, adeoque æquales. Ergo (*per 2 Corol. p. 4. l. 6.*) AB: FH:: BC: HG.

Eodem modo ostendas tangentes ED, KI, & secantes AE, FK esse inter se, ut sunt radii AD, FI.

PROPOSITIO II.

De inventione Sinuum, Tangentium & Secantium.

Invenire alicujus anguli BAC sinum BC, tangentem ED, vel secantem AE, est determinare proportionem quam habent ad suum sinum totum, seu radium circuli AB; id est,

positio quòd AB semidiameter dividatur in quotcunque lubuerit partes, vg. 10000000, ostendere quot hujusmodi partes contineat sinus, tangens, vel secans cujusvis anguli.

Sinuum inquisitio prolixa est & operosa, neque ad institutum nostrum necessaria, cùm passim prostent tabulæ sinuum proportionem exhibentes. Modus quorumvis arcuum & angulorum sinus investigandi videri potest apud P. Tacquet, Geometriæ Pract. libro 1. cap. 2.

Sinubus semel notis, facilè inveniuntur tangentes & secantes iis correspondentes; Nam in triangulis ABC, AED anguli C & D (*per* 4. & 7. *def. h.*) sunt recti, & angulus A communis: Ergo (*per* 2. *Corol. p.* 4. *l.* 6.) Latus AC quod (*per* 34. *l.* 1.) æquale est sinui complementi (ut patebit comparanti DM cum BG in Fig. 1.) est ad BC sinum anguli A; ut radius AD, ad ED tangentem anguli A. Et AC sinus complementi, ad AD sinum totum, ut AB sinus totus, ad AE secantem anguli A.

Corollarium.

His propositionibus nititur constructio tabularum, quarum per Mathesim universam usus est frequentissimus, & vocantur *Sinuum, Tangentium & Secantium Canones* seu *Tabulæ*, quia sinus &c. pro gradibus singulis & minutis computatos exhibent, id est, numeris exprimunt sinuum &c. proportionem ad suam semidiametrum in partes 10000000 divisam. Omittuntur autem sinus &c. minutis secundis & tertiis debiti, tum ne moles in immensum excresceret: tum quia semidiametro in tot partes dissectâ, istiusmodi minutiæ sine dispendio negliguntur. Quòd si quando calculus accuratior desideretur, sinus &c. quibuscunque minutis debitos methodo infrà præscribendâ facilè obtinebis.

Notandum porrò tabulas in duplici esse genere; aliæ vocantur *Naturales* quæ veram proportionem explicant numeris Geometricè proportionalibus: aliæ *Artificiales* quæ naturalium numerorum loco substituunt Logarithmos, seu numeros Arithmeticè proportionales, de quorum inventione & usu, quia alibi dictum est, nil adjicio.

PRO-

PROPOSITIO III.

Minutis quibuscunque datis exhibere sinus ipsis debitos: & è converso notis sinubus, minuta ipsis correspondentia invenire.

Requiratur vg. sinus 30 gr. 5'. 8''. Invento in tabulis, sinu proximè majore, seu sinu 30 gr. 6'. qui est 5015108 & proximè minore, 30 gr. 5', 5012591; quæratur (*per subtractionem*) eorum differentia 2517. Dein fiat ut unum minutum seu 60'' ad 2517, sic 8'' ad aliud. Erit quartus terminus, Fractione spretâ, 335; qui additus sinui 30 gr. 5' dat 5012926 sinum quæsitum gr. 30. 5'. 8''.

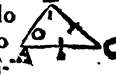
E converso. Si dato sinu aliquo, vg. 2985411, quærantur gradus & minuta ipsi correspondentia; nisi in tabulis compareat ipsissimus sinus datus, inde desumantur sinus proximè minor viz: 17 gr. 22'. seu 2984856, & proximè major 2987632. gr. 17 23'. Quorum differentia est 2776. Deinde ex sinu dato 2985411 subtrahatur proximè minor 2984856 ut habeatur excessus 555. Jam si fiat ut 2776 differentia inter sinus proximos dato, ad 60'': sic 555 excessus sinûs dati supra sinum proximè minorem, ad aliud; quartus terminus 12, indicabit quot minuta secunda illi excessui correspondeant.

Eodem modo reperiuntur sinus quibusvis minutis debiti; & quod dictum est de sinubus, potest similiter tangentibus & secantibus explorandis applicari.

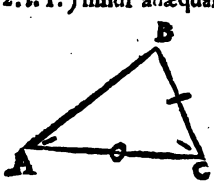
Notanda.

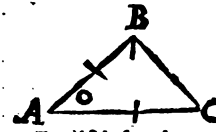
1. Ad trianguli cujuscunque analysim seu resolutionem prærequiruntur tria cognita, viz: duo anguli cum uno latere, vel latus unum cum duobus angulis, vel tria latera, vel demum in sphæricis tres anguli. Dixi *in Sphæricis*, quia in triangulis planis ex solis angulis determinari potest proportio laterum, non tamen (*ut patet ex* 4. *l.* 6.) eorum quantitas absoluta. Termini dati seu cogniti notari solent lineolâ, quæsitus cyphrâ designatur, ut videre est *in Fig. sequenti.*

2. In triangulo aliquo dicitur dari *Oppositio*, quando ex terminis notis duo opponuntur inter se, & tertius opponitur termino quæsito. Unde in triangulo ABC datur oppositio, nam latus notum AC opponitur angulo noto B, & notum latus BC opponitur angulo quæsito A.

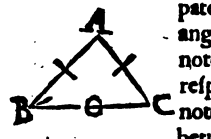


3. Quia omnes anguli cujuſcunque trianguli rectilinei(*per* 32. *l.* 1.) ſimul adæquant duos rectos, hoc ipſo quòd cognoſcantur quivis duo anguli A& C, innoteſcet tertius B, ſubtrahendo ſummam notam angulorum A & C ex duobus rectis, ſeu gradibus 180. ſint vg. angulus A 38 gr. 19′, angulus C 63 gr. 27′. Erit eorum ſumma 101 gr. 46′; quæ ſubtracta ex gr 180 relinquit 78 gr. 14′. quantitatem tertii anguli B. Hinc 1.o Quotieſcunque dantur duo quivis anguli A & C cum quovis latere BC, ſemper habetur oppoſitio; nam ex duobus angulis datis A & C invenitur angulus B oppoſitus lateri quæſito AC. 2.o Dato angulo B, ut inveniatur alter A, ſufficit quòd detur oppoſitio reſpectu tertii C; nam C invento, etiam A notus erit.





Ex dictis ſequitur oppoſitionem tantùm deeſſe quando dantur tria latera ſine angulo, vel duo latera cum uno angulo; qui ſi contineatur inter latera data (*ut in Prop. 3. cap. 3.*) patet quòd non detur oppoſitio: ſi verò ſit angulus B tantùm adjacens alicui lateri noto AB, patet quòd habeatur oppoſitio reſpectu alterius anguli adjacentis C; quo noto, ſcitur etiam angulus A; adeoque habetur oppoſitio reſpectu tertii lateris BC.



4. Quæſtiones Trigonometricæ reſolvuntur per Regulam Auream; adeoque termini ſunt juxta tenorem iſtius Regulæ diſponendi: ita ſcilicet ut primus ſit ejuſdem rationis cum tertio, & ſecundus ejuſdem cum quarto quæſito.

CAPUT II.

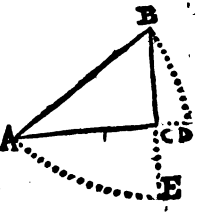
De Analyſi Trianguli rectanguli.

PROPOSITIO I.

In triangulo quovis rectangulo unumquodque latus eſt ſinus anguli ſibi oppoſiti.

Aſſumatur pro radio hypotenuſa AB; & centris A & B

describantur arcus AE, BD, quibus occurrant latera AC, BC producta in D & E.

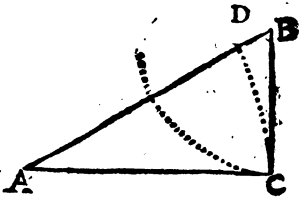


Patet [*ex Constr.*] latus AB esse sinum totum, nam est radius utriusque arcûs. Deinde quia angulus C est (*ex suppos.*) rectus, latus BC ductum à B uno extremo arcûs subtensi BD, est perpendiculare diametro AD transeunti per D alterum extremum ejusdem arcûs: ergo [*per 4. def. h*] BC est sinus anguli oppositi A respectu sinûs totius AB.

Similiter demonstres AC esse sinum anguli oppositi B, respectu ejusdem radii BA seu AB.

PROPOSITIO II.

Si in quovis triangulo rectangulo [ABC] *quodcunque latus* [AC vel BC] *circa angulum rectum* [C] *assumatur pro sinu toto; erit alterum latus tangens anguli sibi oppositi, respectu ejusdem sinûs totius.*



Centro A, radio AC, ducatur arcus DC subtensus angulo A.

Quia angulus C est [*ex suppos.*] rectus, latus BC est perpendiculare ad diametrum AC, & erigitur à C uno extremo arcus subtensi DC, donec in B concurrit cum AB ductâ ex centro A per D alterum extremum ejusdem arcûs subtensi DC: ergo [*per 7. def. h*] BC est tangens anguli sibi oppositi A, respectu radii seu sinûs totius AC. Eodem modo ostendas AC esse tangentem anguli B, respectu radii BC.

Corollarium.

Ex propositionibus explicatis duæ eliciuntur regulæ, quarum ope quivis triangulum rectangulum facilè resolvetur. Prima & principalis vocatur *Regula sinuum*, quia dependet à proportione, seu potiùs identitate, inter trianguli latera & sinus angulorum oppositorum: vocatur etiam *Regula Oppo-*

sitionis, quia locum habet quotiescunque termini propositi inter se opponuntur. Secunda est *Regula Tangentium*, ita dicta quòd nitatur proportione quam habent latera circa angulum rectum ad tangentes angulorum oppositorum. Hæc Regula adhibetur, quoties non intervenit oppositio.

Regula Sinuum.

In omni triangulo rectangulo
Ut quodvis latus, est ad sinum anguli sibi oppositi:
Sic aliud quodvis latus, est ad sinum anguli sibi oppositi.

Et è contrà,

Ut sinus cujusvis anguli, ad latus sibi oppositum:
Sic sinus cujusvis alterius anguli, ad latus sibi oppositum.

Patet ex 1. *p*: *h*. ubi ostensum est unumquodque latus esse sinum anguli sibi oppositi.

Regula Tangentium.

In omni triangulo rectangulo
Ut latus unum circa angulum rectum, ad sinum totum:
Sic aliud latus circa angulum rectum, ad tangentem anguli sibi oppositi.

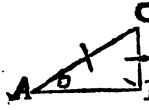
Et è contrà,

Ut sinus totus, ad latus unum circa angulum rectum:
Sic tangens anguli, qui alteri lateri opponitur, ad aliud latus.

Patet ex 2. *p*. *h*, ubi demonstratum est utrumlibet latus circa angulum rectum esse tangentem anguli sibi oppositi respectu alterius lateris circa angulum rectum assumpti in sinum totum.

PROPOSITIO III.

Resolvere triangulum rectangulum, in quo datur oppositio.



1.o In triangulo ABC nota sint latus AC 100 passuum, latus BC 50 passuum, & angulus rectus B: quæratur autem angulus acutus A.

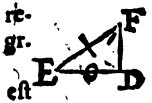
Quia datur oppositio, adhibenda est regula sinuum, & quia quæritur angulus, termini sunt ita disponendi, ut quarto

loco veniat angulus quæsitus A, adeoque secundo loco terminus ejusdem rationis, viz: angulus notus B. Unde primus terminus erit latus AC oppositum angulo noto B, & tertius latus BC oppositum angulo quæsito A. Hæc terminorum dispositio semper est observanda, quando per regulam sinuum quæritur angulus. Fiat itaque

Ut latus AC, 100 *passuum, oppositum angulo noto* B,
Ad sinum anguli noti B, *viz:* 10000000:
Sic latus BC, 50 *passuum oppositum angulo quæsito* A,
Ad sinum anguli quæsiti A.

Peractâ operatione juxta Regulam Proportionis, id est; multiplicato secundo termino [10000000] per tertium [50] & producto [50000000] diviso per primum [100;] quotiens [5000000] erit quartus terminus & sinus anguli quæsiti, qui in tabulis sinuum inventus ostendit angulum A continere gradus 30.

2.° In triangulo EFD dentur angulus D rectus, latus EF 800 pedum, & angulus F 54 gr. 20'. quæratur verò latus ED.

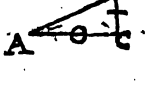


Quia datur oppositio, solutio quærenda est per regulam sinuum; & quia quæritur latus, termini hoc ordine disponentur.

Ut sinus anguli recti D, *seu sinus totus* 10000000
Ad latus oppositum EF 800 *pedum:*
Sic sinus anguli dati F 8124229
Ad latus oppositum & quæsitum ED

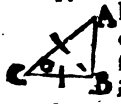
Factâ multiplicatione tertii termini per secundum, & divisione producti per primum; invenitur quartus terminus 649 $\frac{9183200}{10000000}$ Unde concludes latus DE continere pedes 649 $\frac{9}{10}$ circiter.

3.° Datis angulis A & C cum latere BC, si quæratur latus AC neutri angulo dato oppositum; 1° ex angulis notis A & C inveniatur (*ut in Not: 3. c. 1.*) angulus tertius B, quo noto, habetur oppositio; unde 2° per regulam sinuum invenietur latus AC, ut in casu præcedenti.



4.° Notis lateribus AC, BC, cum angulo recto B eorum

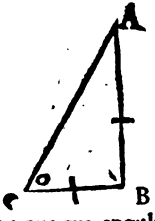
uni opposito; oporteat invenire angulum C neutri notorum



laterum oppositum. Quia datur oppositio respectu anguli A non quæsiti, exploretur per regulam sinuum, (*ut in casu* 1. *h*) angulus A; nam illo invento, [*per Not.* 3. *c.* 1.] notus erit angulus quæsitus C.

Tres supersunt casus, in quibus non datur oppositio sufficiens, ut per regulam oppositionis obtineatur solutio, saltem immediatè, & unicâ operatione. Horum casuum solutionem dabunt Propositiones sequentes.

PROPOSITIO IV.



Datis lateribus (AB, BC) *circa angulum rectum* (B) *invenire quemcunque angulum acutum* (C)

Casus hic solvitur per regulam tangentium, juxta quam sunt semper ita disponendi termini, ut primus sit latus [BC] adjacens angulo quæsito (C;) secundus sinus, totus, seu sinus anguli (B) recti; tertius latus [BA] oppositum angulo (C) quæsito; quartus angulus (C) quæsitus. Sit BC 6000 pedum, BA 10000 pedum. Termini sic stabunt.

Ut latus BC 6000 *pedum*, *Ad sinum totum* 10000000:
Sic latus AB 10000 *pedum*, *Ad tangentem anguli oppositi* C

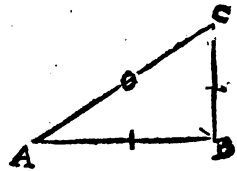
Operare ex præscripto Regulæ aureæ, & deprehendes quartum terminum $16666666\frac{4}{6}$ tangentem anguli quæsiti C; cui in tabulis tangentium quæsitæ proximus invenitur numerus 16664748 tangens anguli 59 gr. 2′, adeoque angulus quæsitus C est 59 gr. 2′. proximè.

Nota per analysim trigonometricam rarò inveniri ipsissimos sinus & tangentes, qui in tabulis habentur; accipi proinde debent quæsitis proximi, neglectâ differentiâ, quæ minuto semper est minor.

PROPOSITIO V.

Iisdem datis, invenire hypotenusam (AC)

Ad casûs hujus solutionem utraque regula adhibetur.

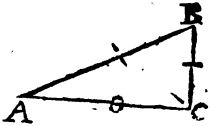


1.° *Per Regulam tangentium, ut in præcedente*, investigatur aliquis angulus acutus (C) quo noto, datur oppositio. Unde

2.° *Per Regulam oppositionis, ut in Prop.* 1. invenitur hypotenusa quæsita AC.

PROPOSITIO VI.

Datâ hypotenusâ (AB) *cum aliquo latere* (CB) *circa angulum rectum* (C) *invenire aliud latus* (AC.)



Solvitur casus hic per *Regulam sinuum bis* adhibitam.

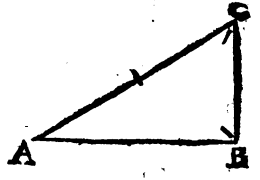
1.° Quæritur angulus A oppositus lateri noto BC & simul invenitur angulus B oppositus lateri quæsito AC.

2.° Ex angulis B & C & latere dato AB, notum fit latus quæsitum AC.

PROPOSITIO VII.

Triangulum resolvere per Logarithmos.

Trianguli resolutio per Logarithmos in eo solo discrepat à praxi hactenus explicatâ, quòd multiplicationis & divisionis loco requirantur additio & subtractio; ut in exemplo constabit.



Sit in triangulo ABC angulus B rectus, angulus C 60 gr. & latus AC pedum 1000: inquiratur autem latus AB

Quia datur oppositio, termini (*per* 1. *h*) sic disponentur.

∠ *Anguli recti* B	*sinus*	*Logarithm.* 100000000

Ad Lateris oppositi AC *Logarith.* 30000000
Sic anguli C *sinus Logarithmicus* 99375306
Ad Logarithmum lateris quæsiti AB.

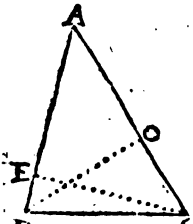
Secundi & tertii termini Logarithmis in unam summam (129375306) per Additionem collectis, subtrahe Logarithmum primi termini; residuum (29375306) est Logarithmus quarti termini, eique in tabulis invento correspondet numerus 866. Est igitur latus AB pedum 866. Idem est Logarithmorum usus, quando per regulam tangentium instituitur operatio.

Nota Logarithmos arcuum & angulorum quæri debere in tabulis Logarithmicis sinuum vel tangentium; Logarithmos verò lateribus rectilineis correspondentes inveniri in tabula Logarithmica numerorum.

CAPUT III.

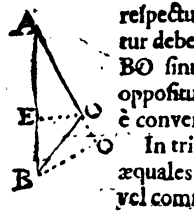
De Analysi trianguli obliquanguli.

PROPOSITIO I.



Trianguli cujusvis rectilinei latera sunt inter se ut sinus angulorum oppositorum; & è converso.

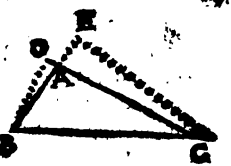
Proponantur triangula ABC. Ab extremis alicujus lateris BC ducantur perpendicula BO, CE ad latera opposita AC, AB, si opus fuerit producta. Erunt in triangulis rectangulis BOC, CEB, (*per p.* 1. *c.* 2.) Latera BO, CE sinus angulorum sibi oppositorum C & B, idque respectu ejusdem sinûs totius BC. Ostendi igitur debet latus AB oppositum angulo C, esse ad BO sinum ejusdem anguli C, sicut latus AC oppositum angulo B ad CE sinum anguli B: & è converso.



In triangulis AEC, AOB, anguli E & O sunt æquales, utpote (*ex constr.*) recti: & angulus A vel communis est ut *Fig.* 1. & 2. vel (*per* 15. *l.* 1.)

utro-

utrobique æqualis, ut in *Fig.* 3. Ergo (*per* 2. *Corol. p.* 4. *l.* 6.) AB : AC :: BO : CE, & (*permutando*) AB : BO :: AC : CE. Ergo (quia similis est in reliqua laterum sinuumque comparatione demonstratio) in omni triangulo obliquangulo eadem est ratio inter latera & sinus angulorum oppositorum.



Observari hîc debet quòd proportio hæc æqualiter subsistat, sive omnes anguli sint acuti, sive aliquis sit obtusus, qualis est angulus BCA *in Fig.* 2.

Corollarium.

Ex hac propositione deducitur pro resolutione trianguli obliquanguli sequens generalis.

REGULA SINUUM SEU OPPOSITIONIS.

In omni triangulo obliquangulo
Ut quodvis latus est, ad sinum anguli sibi oppositi:
Sic quodvis aliud latus est, ad sinum anguli sibi oppositi.

Et à contrà,

Ut sinus cujusvis anguli, ad latus sibi oppositum:
Sic sinus cujusvis alterius anguli, ad latus sibi oppositum.

Notanda.

1. Nomine trianguli obliquanguli hîc intelligimus omne triangulum quod vel non habet, vel nescitur habere aliquem angulum rectum.

2. Diligenter recolenda sunt notanda cap. 1. & quæ dicta sunt Prop. 3. cap. 2. de resolutione trianguli rectanguli habentis oppositionem; nam ista omnia triangulis obliquangulis possunt eodem modo applicari.

PROPOSITIO II.

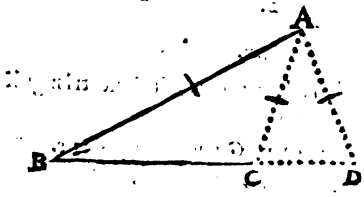
Resolvere triangulum obliquangulum, in quo datur Oppositio.

Quia (ut ostendimus) Regula Sinuum etiam in obliquangulis locum habet, methodus pro rectangulis superiùs (*p.* 3

c. 2.) tradita, est etiam in obliquangulis observanda, quoties intervenit Oppositio; quæ extra duos casus, semper datur sufficienter, & per regulam sinuum, saltem bis adhibitam, eruatur quæsitum. Satis proinde fuerit duos istos casus attingere, in quibus oppositio desideratur. Priùs tamen

Nota 1.° quòd si detur aliquis angulus obtusus, accipi pro eo debeat in operatione, sinus complementi ad duos rectos: vg. Si notus sit angulus 137. gr. 15′; quæri debet in tabulis & in operatione adhiberi sinus 42. gr. 45′.

Nota 2.° quòd si dentur duo latera AB, AC cum angulo adjacente B, casus hic, tametsi interveniat oppositio, determinatam solutionem non admittat, nisi aliunde cognoscatur species anguli C. Nam, quia latus AB & angulus B sunt communia triangulis ABC, ABD, & latera AC, AD possunt esse æqualia; hæc non sufficiunt ad determinandum utrùm tertium latus sit BC, an verò BD; nec quales sint reliqui anguli.



Quòd si aliunde scias speciem anguli C, difficultas evanescit; nam in triangulo minore ABC semper est obtusus angulus C, in majore ADB semper est acutus D. quod sic ostenditur.

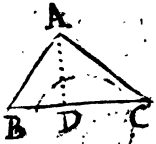
In triangulo ACD latera AC, AD sunt (*ex suppos.*) æqualia: ergo (*per* 5. *l.* 1.) æquantur anguli ACD, ADC: ergo, cùm simul sint (*per* 32. *l.* 1.) minores duobus rectis, singuli sunt acuti: ergo angulus D est acutus; &, quia ACD est etiam acutus, angulus ACB (*per* 13. *l.* 1.) est obtusus.

PROPOSITIO III.

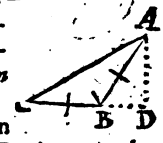
Datis duobus lateribus (AB, BC) *cum angulo comprehenso* (B) *invenire tertium latus* (AC) *& reliquos angulos* (A & C).

Ab angulo aliquo ignoto A dimitte perpendiculum AD

ad latus oppositum BC, si opus sit, productum. Sic fient duo triangula BAD, CAD rectangula in D.



Quoniam in triangulo ABD, noti sunt anguli, D (*ex constr.*) rectus, & ABD, qui vel idem est cum dato ABC, vel (*per 13. l. 1.*) dati complementum ad duos rectos; notus est etiam tertius angulus BAD. Ex omnibus angulis & latere dato AB (*per regulam sinuum*) inquirantur latera AD & BD.



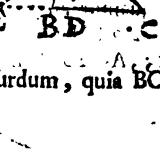
Invento BD, scitur etiam DC: ergo in triangulo ACD habentur latera duo AD, CD circa angulum D (*ex constr.*) rectum. Ex quibus (*per regulam tangentium*) noti fiunt anguli ACD, & CAD; & ex his omnibus (*per regulam sinuum*) reperitur latus AC.

Nota 1.° quòd si angulus datus ABC sit obtusus, accipi debeat in prima operatione angulus ABD ejus complementum ad duos rectos.

Nota 2.° quòd si inveniatur latus BD minus latere BC dato, perpendiculum cadat intrà; aliàs extra triangulum. Hoc diligenter est observandum ut constet utrùm additione, an subtractione opus sit, ut ex inventis eliciantur quæsita.

LEMMA I.

Si ad trianguli (ABC) *latus maximum* (BC) *ab angulo opposito* (A) *ducatur perpendiculum* (AD) *cadet intra triangulum.*

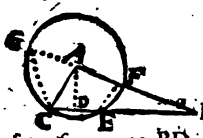


Si neges, cadat extra ad E; erit angulus E rectus: ergo (*per 47. l. 1.*) quadratum AC æquatur quadratis AE, EC adeoque majus est quadrato BC: ergo linea AC est major lineâ BC. quod est absurdum, quia BC supponitur latus maximum trianguli ABC.

LEMMA II.

Si ad trianguli (ABC) *latus maximum* (BC) *demittatur perpendiculum* (AD,) *erit basis* (BC) *ad aggregatum reliquorum laterum* (AB, AC:) *ut differentia horum laterum, ad differentiam segmentorum baseos.*

Centro A, intervallo minimi lateris AC, describatur circulus, cui occurrat in G, latus BA productum, quia æqualia sunt AC, AG, AF, patet BG esse aggregatum laterum AB, AC, & lineam BF esse eorundem differentiam. Item quia (*per 3. l. 3.*) æqualia sunt ED, DC, patet BE esse differentiam inter baseos segmenta BD, DC. Ostendi igitur debet quòd sit BC: BG :: BF : BE. Junge puncta G, C, & F, E.

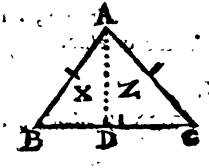


In triangulis GBC, EBF angulus B est communis, & angulus G cum FEC (*per 22. l. 3.*) æquatur duobus rectis. Sed FEC cum FEB (*per 13. l. 1.*) facit duos rectos: ergo G & FEB sunt æquales: ergo (*per 2. Corol. p. 4. l. 6.*) BC: BG:: BF: BE.

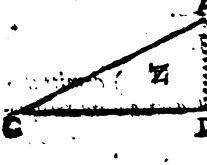
PROPOSITIO IV.

Datis tribus lateribus investigare angulos.

1.o Si aliqua duo latera AB, AC sint æqualia, ab angulo A inter illa comprehensò, ad basim BC bisectam in D, ducatur perpendiculum AD. Sic fient duo triangula X & Z sibi mutuò æquilatera, ut patet: & quia AB, AC ponuntur æqualia, anguli oppositi ADB, ADC (*per 8. l. 1.*) sunt æquales, adeoque recti: ergo in triangulo X cognoscuntur latera AB & BD dimidium dati BC, cum angulo recto D. Ex his, (*per regulam sinuum*) explorabitur angulus BAD, adeòque & ABD, quibus (*per 8. l. 1.*) æquantur anguli CAD, ACD.



2.o Si latera omnia sint inæqualia, vg. BC pedum 21, AB 33, AC 20. demisso ab angulo A ad latus maximum BC, perpendiculo AD; erit ut basis BC (seu 21) ad aggregatum laterum AB, AC [seu 33: 20] sic horum laterum differentia (seu 7) ad differentiam segmentorum BD, DC. Ductis igitur 7 in 33, & producto 231 diviso

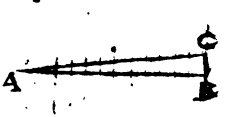


per 21, quotientem 11 aufer à basi BC 21. Residui dimidium 5 erit segmentum minus DB, adeoque segmentum majus D est 16. Ergo in triangulo Z nota sunt latera AC, DC, cum angulo recto ADC, & in triangulo X latera AB, BD cum angulo recto ADB; ex his (*per regulam sinuum*) innotescent anguli A, B, C.

Nota ea omnia, quæ [*Prop. 7. cap. 2.*] *dicta sunt de usu Logarithmorum in resolutione trianguli rectanguli, esse etiam de obliquangulis intelligenda.*

Hic etiam notari debet quòd si, multiplicatis inter se secundo & tertio termino, prodeat numerus minor primo termino; quæsitum sit Fractio, cujus numerator est productus inventus, denominator verò primus terminus.

Exemplum. Dentur in triangulo ABC, angulus B rectus, ejusque sinus 10000000; latus AC 8 pedum; & angulus A 5 gr. 44'. 21", cujus sinus est 1000000; quæratur autem latus BC. Secundo termino 8, ducto in tertium 1000000, habetur Productus 8000000 minor primo termino 10000000; unde latus quæsitum BC est minus pede, seu Fractio pedis quæ sic exprimitur $\frac{8000000}{10000000}$ vel sic $\frac{8}{10}$ assumpto nimirum in denominatorem primo termino, in numeratorem Producto inventa.



Si, dum per Logarithmos instituitur operatio, summa secundi & tertii termini sit minor primo; erit quartus terminus Fractio, cujus denominator est numerus correspondens Logarithmo primi termini, numerator verò numerus correspondens summæ per secundi & tertii termini additionem inventæ.

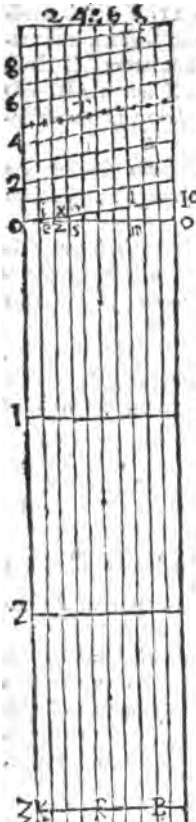
CAPUT IV.

De analysi triangulorum per scalam & quadrantem.

PROPOSITIO I.

Constructio Scalæ Geometricæ.

Nomine *Scalæ* hîc intelligitur regula quævis in 10 partes



æquales divisa secundùm longitudinem, lineis 0. 1. 2. 3. &c. à latere ad latus ductis; & in 10 partes æquales secundùm latitudinem, lineis ductis ab extremo ad extremum. Subdividitur etiam suprema decima pars in alias 10 partes æquales, per lineas obliquas & parallelas, 0. 2. 4. 6. &c. Demum lineis longioribus intermediis adscribuntur in regulæ extremo notæ, 2, 4, 6 &c. & perfecta est scala.

Tota scalæ longitudo accipitur pro 1000 partibus, unde pro pluries 1000 partibus est pluries repetenda. Pro centenis assignantur novem decimæ partes scalæ. Pro decadibus relinquitur una decima pars in suas decades distributa. Numerantur unitates in lineis quæ unitatum notis designantur.

Ex: gr: Ko exhibet partes 300: K8 partes 380: RT partes 354: *e i* unam partem: *xz* duas: *sn* tres: *lm* septem &c.

Quòd partes centenæ & decades rectè numerentur, patet ex constructione scalæ; quòd autem rectè numerentur unitates, sic ostendo.

In triangulo *lo, o, o* ducitur latus *i e* parallelum [*ex constr.*] lateri *lo, o*: ergo (*per* 1. *Coroll. p.* 4. *l.* 6.) *ie: lo, o:: oe: oo.* sed *oe* est (ex constr.) decima pars rectæ *oo*: ergo *ie* est decima pars rectæ *lo, o*. Eodem modo probatur *xz* esse $\frac{2}{10}$, *ns* $\frac{3}{10}$ *lm* $\frac{7}{10}$ ejusdem rectæ *lo*, & sic de cæteris.

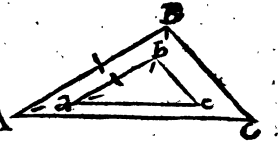
PROPOSITIO II.

Resolvere quodcunque triangulum per Scalam & Quadrantem.

Usus Scalæ & Quadrantis in triangulorum resolutione ad quatuor casus reducitur; nec refert utrùm triangulum propositum sit rectangulum an verò obliquangulum; nam eadem in utroque methodus observatur.

Primus Casus.

Ex aliquo latere (AB) *& duobus angulis* (A & B) *reliqua latera invenire.*

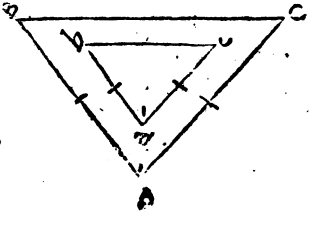


Contineat latus datum AB pedes 143. Angulus A 36 gr. 9′. Angulus B 99 gr. 12′. Desume ex scala lineam *a b* dato lateri AB similem, id est, continentem tot scalæ partes quot pedes continet AB. Dein lineâ *a b* in chartam translatâ (*per schol.* 23. *l.* 1.) effice angulos *a* & *b* æquales datis A & B, & perfice triangulum *abc* quod (*per* 2. *Corol. p.* 4. *l.* 1.) erit simile dato ABC. ergo (*per* 4. *l.* 6.) *ab*: AB :: *bc*: BC, & *ab*: AB :: *ac*: AC. ergo etiam BC, AC tot pedes continent, quot scalæ partes continent *bc*, *ac*. Has igitur circino exceptas scalæ admove, & innotescent latera BC, AC. viz: BC 120 pedum: AC 201 pedum.

Nota perinde esse quinam duo anguli dentur; nam ex quibusvis duobus datis tertius cognoscitur.

Secundus Casus.

Ex duobus lateribus [AB, AC] *& angulo contento* [A] *invenire aliud latus* [BC] *& reliquos angulos.*



Accipiantur ex scala, *ab*, *ac* similes datis AB, AC; factoque (*per schol.*

23. *l.* 1.) angulo *bac* æquali dato BAC, jungantur puncta *b* & *c*. Erunt (*per* 6. *l.* 6.) triangula *abc*, ABC similia; adeoque *ab*: AB:: *bc*: BC. Ergo *bc* scalæ applicato invenietur latus BC quæsitum.

Deinde quia latera triangulorum *abc*, ABC sunt similia, anguli B, *b*, & C, *c*, oppositi similibus lateribus AC, *ac*, & AB, *ab* (*per* 5. *l.* 6.) sunt æquales. Inventis igitur [*per schol. p.* 23. *l.* 1.] angulis *b* & *c*, noti erunt anguli quæsiti B & C.

Tertius Casus.

Datis omnibus lateribus invenire angulos.

Si ex scala desumas tres lineas similes lateribus datis, triangulum ex eis [*per* 22. *l.* 1.] constitutum, erit (*per* 5. *l.* 6.) æquiangulum triangulo dato. Unde si trianguli istius angulos explores [*per schol. p.* 23. *l.* 1.] inde cognoscentur anguli quæsiti.

Quartus Casus.

Datis duobus lateribus cum aliquo angulo adjacente, reliqua invenire.

Hic est casus ille indeterminatus de quo dictum est Prop. 2. cap. 3. Vide Figuram istic positam, cui si ex scalæ partibus similem confeceris, erunt (*per* 5. *l.* 6.) triangula æquiangula, & latera proportionalia: adeoque tertium latus BC vel BD, & anguli non dati, modo superiùs tradito, noti fient. Casus hic infrequens est, & ad trianguli analysim ferè inutilis; unde non est cur illi diutiùs immoremur.

CAPUT V.

Longitudinum & Altitudinum Dimensio.

Trigonometriæ Planæ hactenùs explicatæ ut usus constet, praxisque evadat facilior, problemata Geometrica adjungere visum est; pauca quidem pro rerum varietate, talia tamen, ut præcipuam scientiæ illius partem complectantur.

Distan-

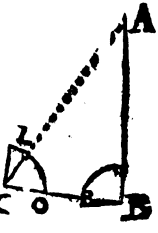
Distantiarum seu Longitudinum Horizontalium investigatio.

PROBLEMA I.

Metiri distantiam ex una tantùm parte accessibilem.

PRIMUS MODUS.

Sit talis distantia BA, latitudo Fluminis vg. accessibilis tantùm in B. eligatur aliqua alia statio C, & mensuretur mechanicè stationum intervallum CB, quod oportet modiocre esse respectu distantiæ quæsitæ BA. Dein quadrante explorentur anguli C & B; ex quibus & latere CB notis, invenietur (*per.* 2. *c.* 3. *b.*) latus BA Fluvii latitudo quæsita.

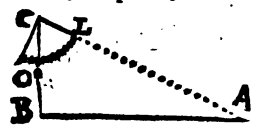


Nota 1.° Ad intervallum aliquod mechanicè mensurandum adhiberi posse vel catenulas ferreas, vel decempedam ligneam, vel aliud ejusmodi instrumentum. Inter mensurandum autem curare debet mensor ut pergat in directum, sequendo semper eandem lineam rectam; ad quod juvabunt duo baculi modico intervallo distantes, erecti vg. in C & O, à quibus ita retrocedat mensor, ut oculus ejus constanter maneat in eadem linea cum utroque baculo.

Nota 2.° Hujusmodi trianguli, quod scil: vel horizantale est, vel ad horizontem inclinatur, angulos exquiri posse ope quadrantis vel semicirculi, circa cujus centrum sit regula mobilis dioptris instructa. Ex: gr: Si requiratur angulus ACB; in puncto C ita statue centrum quadrantis, ut per dioptras regulæ superimpositæ ejus lateri CO, conspiciatur punctum B. Dein, quadrante manente in eodem situ, moveatur regula donec per dioptras appareat punctum A; & erit arcus LO, cujus gradus notantur in margine instrumenti, mensura anguli quæsiti ACB.

Nota 3.° Praxim explicatam adhiberi perinde posse, sive distantia mensuranda sit horizonti parallela, sive ad illum inclinata.

SECUNDUS MODUS.



Sit, ut priùs, BA latitudo Fluminis; BC altitudo oculi menſoris. In triangulo ABC angulus B eſt rectus, angulus C invenietur ope quadrantis, & latus BC vel notum eſt, vel facilè innoteſcet. Ex his tribus (*per. 3. c. 2. h*) notum fiet latus BA, diſtantia quæſita.

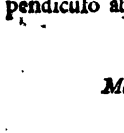
Nota 1.o Si diſtantia propoſita BA ſit magna, expedire ut menſor conſcendat locum editum, tumulum vg. vel turrim, cujus nota ſit altitudo.

Nota 2.o Angulum ACB [idemque eſt in omnibus triangulis, quorum plana ſunt horizonti perpendicularia] inquiri debere quadrante accurato, à cujus centro C dependeat perpendiculum CO. Nam ſi quadrantis centrum C ita admoveas oculo, ut per latus CL conſpicias punctum A, arcus LO à perpendiculo abſciſſus, indicabit quantitatem Anguli C.

PROBLEMA II.

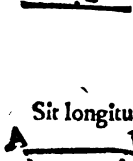
Metiri longitudinem penitùs inacceſſam.

PRIMUS CASUS.



Proponatur diſtantia inacceſſa BA jacens in directum reſpectu menſoris conſtituti in C. erit CBA linea recta, & CB, CA diſtantiæ acceſſibiles in C; quibus [*per Probl: præced:*] inventis, ex CA ſubtrahatur CB, eritque reſiduum BA diſtantia quæſita.

SECUNDUS CASUS.

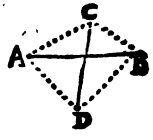


Sit longitudo propoſita BA, obverſa menſori exiſtenti in C. Erunt CA, CB duæ diſtantiæ acceſſibiles in C; quas ſi [*per Probl: præced:*] inveſtigaveris, notumque feceris quadrante vel ſemicirculo angulum C; cognoſcentur in triangulo ACB latera CA, CB, & angulus contentus C. Ex his (*per. 3. c. 3. h*) invenietur longitudo quæſita BA.

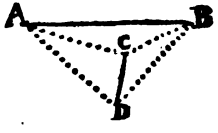
PROBLEMA III.

Duarum Urbium diſtantiam inquirere.

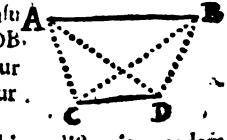
Sint duæ civitates A & B ex planitie vel intermediâ vel adjacente conſpicuæ, in qua eligantur duæ ſtationes C & D, quarum intervallum CD menſuretur vel mechanicè, vel *per Probl.* 1.



Deinde exploratis in ſtatione C angulis ACD, BCD, & in ſtatione D angulis CDA, CDB [quod fieri debet, ſi diſtantia AB ſit notabilis, quadrante magno & exquiſito;] quæratur reſolvendo triangulum ADC, latus AD; &, reſolvendo triangulum BDC, latus BD.



Denique notis in triangulo ADB lateribus AD, BD, cum angulo comprehenſo ADB [qui in 1.o & 2.o caſu conflatur ex angulis CDA, CDB priùs inventis, & in 3.o caſu habetur ſubtrahendo CDA ex CDB] invenitur latus AB diſtantia quæſita.



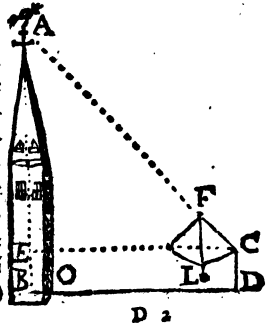
Quod dictum eſt de duarum Urbium diſtantia, eodem modo applicatur quorumvis locorum intervallo.

Altitudinum menſura.

PROBLEMA IV.

Metiri altitudinem acceſſibilem in Plano poſitam.

Sit AB altitudo menſuranda; Menſor in D, ejuſque oculus in C. Ex C intelligatur duci recta CE parallela & æqualis lineæ horizontali DB; & erit in triangulo ACE angulus E rectus, utpote æqualis (*per.* 27. *l.* 1.) angulo B (*ex ſuppoſ.*) recto. Deinde quia lineæ FL, AB ſunt horizonti perpendiculares, adeoque (*per.* 6. *l.* 11.) parallelæ, anguli A & CFL ſunt (*per.* 27. *l.* 1.)



æquales; invento igitur per quadrantem CFL, notus erit angulus A. Denique latus CE, seu DB cognoscetur modo aliquo superiùs explicato. Ex his tribus reperietur (*per.* 3. *c.* 2. *h*) latus AE; cui addatur EB seu CD altitudo oculi mensoris, & nota erit AB altitudo quæsita.

Assumpsi lineas CE, DB, item CD, ED esse æquales, quod sic ostendo. Quia CD, EB sunt perpendiculares horizonti, sunt (*per.* 6. *l.* 11.) parallelæ. Sed etiam CE, DB sunt (*ex suppos.*) parallelæ: ergo (*per.* 35. *def. l.* 1.) DBEC est parallelogrammum: ergo (*per.* 34. *l.* 1.) latera ejus opposita CE, DB, & CD, EB sunt æqualia.

Nota quòd, si explorans distantiæ angulum CAE collimes in aliquod punctum A unde demissum perpendiculum AB cadit intra ipsam turrim; tunc distantiæ DO, mechanicè vel aliter notæ, adjicienda sit linea OB, quæ inter perpendiculum & exteriorem turris superficiem intercipitur.

PROBLEMA V.

Altitudinem accessibilem sine trianguli resolutione invenire.

Quia non semper præsto sunt tabulæ, nec vacat semper analysim Trigonometricam adhibere, subjicio alios modos faciles, quibus obviæ altitudines inquirantur.

PRIMUS MODUS. *Vide fig: prac:*

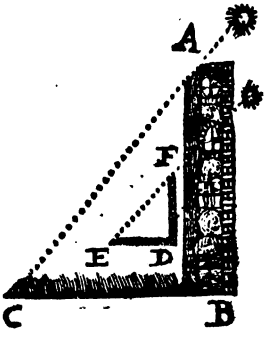
Per quadrantis dioptras collimans in Turris AB fastigium A, accede vel recede donec angulus CFL, adeoque & angulus A sit 45 gr. Tunc metire distantiam CE, eique adjice CD seu EB; eritque summa AB altitudo quæsita.

Nam in triangulo ACE angulus E est rectus: ergo (*per* 1. *Corol. p.* 32. *l.* 1.) anguli A & C simul adæquant unum rectum. Cùm igitur A sit 45 gr. anguli A & C sunt æquales: ergo (*per* 6. *l.* 1.) æquantur latera AB, CE. Cætera per se patent.

SECUNDUS MODUS.

Baculum FD in æquales partes divisum erige perpendiculariter in D; & erunt, sole lucente, duo triangula similia ABC, FDE. Nam anguli B & D sunt recti, C & E sunt

anguli ejusdem altitudinis solis: ergo (*per* 2. *Corol. p.* 4. *l.* 6.) ut ED umbra baculi, est ad FD baculi longitudinem: sic BC umbra turris, ad AB turris altitudinem.



Idem facilius obtinebis metiendo turris umbram quando sol altus est 45 gr. Tunc enim turris umbra, & altitudo sunt æquales, ut ex dictis patet. Hæc autem solis altitudo observari potest Quadrante vel Analemmate; vel [si ista desint] describendo in plano horizontali circulum, in cujus medio erigatur stylus radio æqualis, planoque circuli rectus; nam styli hujus umbra attinget præcisè circumferentiam circuli, quando sol 45 gradus supra horizontem attollitur.

PROBLEMA VI.

Metiri altitudinem accessibilem, quamvis non sit in plano.

Sit Turris AB (vide *Fig.* 2. *Probl.* 10.) in qua si lapidum junctura non sint horizontales, ducatur linea horizontalis CD, quæ mensuretur mechanicè, sicut & altitudo IC. Dein ex C observetur quadrante angulus CAD, ex quo & angulo recto D, & latere DC notis, invenitur (*ut suprà*) AD; cui addita CI vel DB, dat AB altitudinem quæsitam.

Eodem ferè modo Mensor in turris fastigio positus inveniet illius altitudinem; eademque praxis inservire potest ad explorandam, fossæ vg. profunditatem.

PROBLEMA VII.

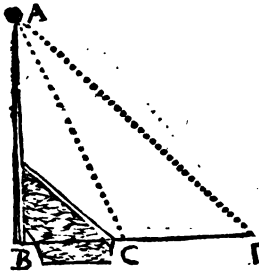
Metiri altitudinem inaccessam.

PRIMUS MODUS.

Per *Probl.* 2. exploret Mensor suam ab altitudine prei posita distantiam; & eâ notâ, *per Probl.* 4. vel 5. ipsam altitudinem.

SECUNDUS MODUS.

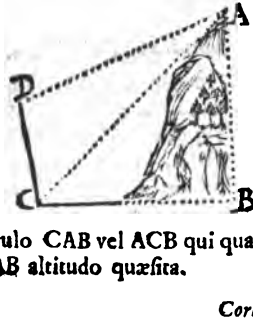
Invento in ſtatione C angulo CAB, retrocede ad diſtantiam ſatis magnam (ut ſit inſtrumento perceptibilis differentia angulorum CAB, DAB) *vg.* ad paſſus 150, in D: & in D, notum fac angulum DAB.



Jam quia angulus B eſt rectus, patet (*ex 2. c. 2. b*) lineas CB, DB eſſe, reſpectu ſinûs totius AB, tangentes angulorum CAB, DAB. Horum igitur tangentibus quæſitis, fiat ut DC tangentium differentia, ad AB ſinum totum: ſic paſſus 150, ad aliud; & erit quartus terminus altitudo quæſita AB.

TERTIUS MODUS.

Sit alicujus Montis altitudo perpendicularis AB intra ipſius viſcera inacceſſibilis & inviſibilis. Eligantur duæ ſtationes collaterales C & D, quarum intervallum CD, ſicut & anguli DCA & D inquirantur ut ſuprà. His notis in triangulo DAC, inveniatur latus AC. Deinde in triangulo CAB, ex angulo recto B, & latere oppoſito AC, & angulo CAB vel ACB qui quadrante explorabitur, inveſtigetur AB altitudo quæſita.



Corollaria.

1. Ex iiſdem notis in triangulo CAB innoteſcet latus CB, diſtantia Menſoris ab AB perpendiculari montis altitudine.

2. Eodem modo metiaris nubis altitudinem: debent tamen, propter nubis mobilitatem, eſſe duo Menſores; unus in C, alter in D, qui ſimul, dato ſigno, in nubem collimantes, obſervent angulos C & D.

Notandum.

Nisi summa adhibeatur cura, facilè erratur in angulis investigandis, unde in reliquam operationem error frequenter derivatur. Ad incommodum hoc avertendum, duo præcipuè conferent; Primum, si in instrumentorum delectu & usu magna adsit diligentia: Secundum, si assumatur stationum intervallum ita proportionatum longitudini mensurandæ, ut anguli fiant sensibiles. Hinc stationes collaterales aliis præstant, quia minori indigent intervallo, ut angulorum quantitas determinetur. Hinc etiam praxis ultima, quâ montis altitudinem investigavimus, videtur reliquis anteferenda.

PROBLEMA VIII.

Distantias & altitudines per Scalam Geometricam invenire.

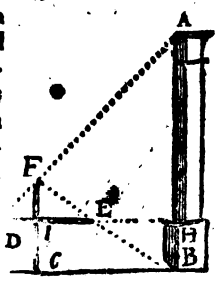
Longitudines hactenus investigavimus resolvendo triangulum aliquod, cujus nota fuerunt vel duo latera cum uno angulo, vel anguli duo cum aliquo latere. Si idem triangulum, quodcunque sit, [illegible]olvatur methodo explicatâ *Cap.* 4. eadem Problematis solutio per scalam obtinebitur.

PROBLEMA IX.

Idem præstare per Crucem Geometricam.

Crux Geometrica (quæ vulgò *Baculus Jacobæus* dicitur) constat duabus Regulis DE, FC in æquales partes divisis, & ita inter se aptatis, ut (sive brachium DE moveatur sursùm, sive deorsùm, sivè protrudatur, sive retrahatur) semper efficiant angulum rectum I.

PRIMA PARS.



Sit mensuranda distantia CB. In C erigatur perpendiculariter (quod fit applicando normam vel quadrantem lateribus instrumenti) regula FC: alterum verò brachium DE impellatur vel retrahatur, donec per extrema F & E appareat punctum B. Sic fient duo triangula FIE, FCB habentia angulum ad F communem, & angulos FIE, FCB (*ex constr.*) rectos, adeoque

æquales. Ergo (*per* 2. *Corol. p.* 4. *l.* 6.) FI: IE:: FC: CB. Ubi tres primi termini noti sunt in partibus instrumenti: ergo quartus CB, seu distantia quæsita, *per Reg: Prop:* nota fiet in iisdem partibus.

SECUNDA PARS.

Quæratur columnæ altitudo AB. Instrumento erecto, ut priùs, perpendiculariter in C, inventâque distantiâ interjectâ CB; elevetur vel deprimatur brachium DE, donec per extrema D & F conspiciatur fastigium A, & intelligatur DE produci donec occurrat columnæ in H. Sic fient duo triangula DIF, DHA; quibus communis est angulus D, & anguli DIF, DHA (*per.* 27. *l.* 1.) æquales propter parallelismum laterum FC, AB. Ergo (*per* 2. *Corol. p.* 4. *l.* 6.) DI: IF:: DH: HA. Ubi duo primi termini cognoscuntur in partibus instrumenti, tertius verò DH habetur addendo DI, distantiæ IH vel CB priùs in iisdem partibus inventæ. Ex his tribus invenitur HA, cui addita HB, seu [illegible], dat altitudinem quæsitam AB.

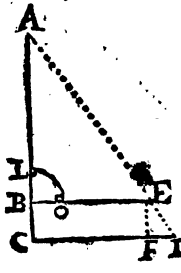
PROBLEMA X.

Accessibilem distantiam vel altitudinem aliter sine tabulis metiri.

PRIMA PARS.

Sit distantia proposita AB, Fluvii vg. latitudo, accessibilis in B.

In statione B erige baculum, à quo ita recede ad distantiam modicam in C, ut punctum A & baculum B videas in eadem linea rectâ, & in C erige secundum baculum. Deinde centro quadrantis ita collocato in prima statione B ut per unum illius latus BL videas punctum A; per aliud latus BO observa aliquod punctum E, ubi figatur tertius baculus. Denique à secunda statione C digredere (similiter ope quadrantis) ad angulos rectos, donec in aliquo loco D appareant punctum A & baculus E in linea rectâ;

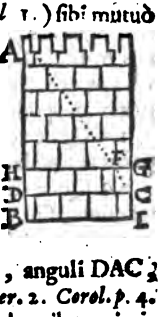


recta; tunc in Distatue quartum baculum. Baculis sic dispositis, metire mechanicè tria intervalla CD, CB, BE. Quibus inventis, vg. CD 140 pedum, BE 115. CB 34; fiat ut excessus intervalli CD supra BE, ad CB: sic BE ad aliud, (id est, ut 25 ad 34, ita 115 ad aliud;) & erit quartus terminus (pedes $156\frac{2}{5}$) BA Fluvii latitudo quæsita.

Demonstr. Ducatur EF parallela lineæ AC. Quia anguli ABE, ACD sunt (*ex constr.*) recti, adeoque æquales, lineæ BE, CD sunt (*per. 29. l. 1.*) parallelæ. Sed etiam AC seu BC & EF sunt (*ex constr.*) parallelæ: ergo (*per. 34. l. 1.*) latera opposita BC, EF, & BE, GF sunt æqualia: ergo FD est differentia intervallorum BE, CD. Deinde, quia AC, EF sunt parallelæ, æquantur (*per. 27. l. 1.*) anguli A & FED; & quia BE, CD sunt parallelæ, æquales sunt (*per eandem*) anguli AEB, EDF. Ergo (*per. 2. Corol. p. 4. l. 6.*) FD: EF seu BC:: BE: BA.

SECUNDA PARS.

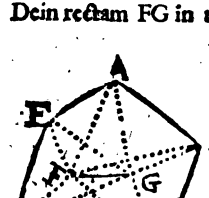
Sit mensuranda altitudo Turris AB; in qua si lapidum commissuræ non sint horizontales, ducantur modico intervallo lineæ CD, GH horizonti, adeoque (*per 30. l. 1.*) sibi mutuò parallelæ. Deinde oculo in C constituto collima in turris angulum A, & nota punctum F quo radius visualis CA secat commissuram lapidum seu lineam superiorem GH. Sic habebis duo triangula similia GFC, ACD; nam quia HG, DC sunt parallelæ, alterni GFC, FCD seu ACD (*per. 27. l. 1.*) sunt æquales: & quia AB, GI sunt (*ex Regulis Architectonices*) horizonti perpendiculares, adeoque (*per 6. l. 11.*) parallelæ, anguli DAC, FCG sunt (*per. 27. l. 1.*) æquales: ergo (*per. 2. Corol. p. 4. l. 6.*) GF: GC:: CD: AD. Mensuratis proinde tribus primis terminis, cognoscetur (*per Regulam auream*) quartus AD; cui adjiciatur BD, & nota erit AB altitudo quæsita.



PROBLEMA XI.

Agri vel cujusvis Figuræ rectilineæ Ichnographiam describere, & dimensiones invenire.

In agro duas tales stationes elige, ut rectæ per eas ductæ in nullum agri angulum incurrat; & mensurato mechanicè, vel aliter, stationum intervallo, ex scala desumatur recta FG continens tot scalæ partes, quot passus vel pedes continet illud intervallum.



Dein rectam FG in tabulam transfer, quam statue horizontaliter in alterutra stratione, ita ut lineæ FG unum extremum F immineat dictæ stationi, & aliud extremum G in alteram stationem B dirigatur ope regulæ dioptris instructæ. Tunc regulam dirigens à puncto F ad singulos agri angulos, duc versùs eos per latus regulæ, rectas FA, FB, FC, FD, FE.

Ad eundem modum collocatâ tabulâ horizontaliter in secunda statione, ita ut G immineat huic stationi, & F in alteram dirigatur; ex G ducantur versùs singulos agri angulos rectæ occurrentes prioribus in punctis A, B, C, D, E, quæ jungantur lineis AB, BC, CD, DE, EA; & erit figura ABCDE Ichnographia quæsita, agro seu plano proposito omnino similis. Unde

1.° Quot scalæ partes continet quævis linea in tabula, tot passus vel pedes continet agri latus correspondens.

2.° Quot scalæ partes quadratas continet Ichnographia, tot sunt passus vel pedes quadrati in agro mensurando.

3.° Anguli in tabula æquantur angulis in agro correspondentibus; unde (*per schol. 23. l. 1.*) ex tabula quilibet agri angulus cognoscetur.

Ratio horum omnium est, quòd triangula quæ in tabula describuntur, sint (ut patet *ex constr.*) similia totidem triangulis in agro; adeoque omnes lineæ in tabula sunt ad agri latera correspondentia, sicut FG ad stationum intervallum. Sed FG (*ex constr.*) continet præcisè tot scalæ partes, quot pedes vel passus continet stationum distantia: ergo &c.

CAPUT VI.

Inveſtigantur Diameter & Circumferentia Sphæræ Terreſtris: item diſtantiæ Solis & Lunæ à Terra, earumque diametri.

Legantur ex Euclidis libro 3. Prop. 18. 20. 21. 22. 31. 32. 36 cum Corollariis.

Ex libro 5. Prop. 7. 9. 11.

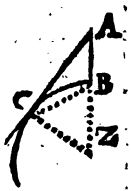
Ex libro 6. Prop. 14. 16. 17. cum Corollariis.

PROBLEMA I.

Terræ diametrum invenire.

Eligatur juxta mare mons excelſus, cujus altitudo perpendicularis ſit AB, quæ producta tranſit per terræ centrum Z: ſit etiam AC radius viſualis horizontem ſtringens, id eſt ſphæram terreſtrem tangens in C.

Quadrante magno & accurato exploretur angulus A; quod ut ſecuriùs fiat, captetur ortus vel occaſus alicujus ſtellæ, ſeu potiùs Solis vel Lunæ, quando extremus tantùm aſtri limbus ſupra horizontem eminet. Nam tunc collimans in aſtrum, ſimul collimabis in aliquod horizontis punctum C. Quæratur etiam (*per Probl. 7. c. 5.*) montis altitudo AB, quod fiat quàm accuratiſſimè.



Noto in triangulo ACZ angulo A, quia angulus C (*per 18. l. 3.*) eſt rectus, notus etiam erit (*per. 4. Corol. p. 32. l. 1.*) angulus Z: ergo (*per idem*) nota eſt in triangulo CZB ſumma angulorum CBZ, BCZ; qui cùm, ob laterum ZB, ZC æqualitatem, ſint (*per. 5. l. 1.*) æquales, utriuſque ſeorſim quantitas cognoſcetur. Subtrahe jam angulum ZCB ex recto ZCA, & reſtabit angulus BCA notus. Adeoque in triangulo CBA noti ſunt anguli A, BCA, & *per. 4. Corol. p. 32. l. 1.*) angulus ABC, cum latere AB, ex quibus invenietur latus AC.

Denique in triangulo ACZ, ex latere AC & omnibus angulis deprehenditur latus C Z terræ ſemidiameter, quæ duplicata dat diametrum quæſitam.

Notanda.

1 Supponit hoc Problema quòd Terra, seu moles Terræ quæ, habeat figuram Sphæricam; quæ veritas variis modis solet demonstrari. 1.° Ex umbra terræ, quæ in Eclipsi Lunari semper apparet circularis. 2.° Quia ubicunque fueris, horizon visibilis circulo terminatur. 3.° Quia montium sublimiorum vertices soli à longè conspiciuntur, accedentibus verò detegúntur paulatim, idque eodem modo à quacunque tandem parte accesseris. 4.° Ex siderum motu, ortu & occasu respectu locorum diversorum, aliisque cæli phænomenis, quæ (ut patebit consideranti) quamcunque figuram non Sphæricam terræ attribueris, fierent longè aliter, quàm de facto contingunt.

2. Supposuimus hîc, quod & superiùs sæpè, centrum gravium idem esse cum centro terræ. Hoc autem est ab experientia manifestum; nam videmus gravia ubivis locorum ferri deorsum motu ad horizontem perpendiculari, & ædium parietes citò collabi, si à situ perpendiculari declinent. Hæc fusiùs explicata vide apud Ricciolum *in Almagesto. libro 2. cap.* 1. & 4.

3. Supposuimus *cap.* 5. in altitudinum inquisitione angulum ad horizontem CBA esse rectum; & meritò, quia in distantiâ non valdè magnâ differentia est insensibilis. Est tamen revera recto major; nam cum angulo acuto CBZ adæquat (*per.* 13. *l.* 1.) duos rectos. Habetur autem justa mensura anguli CBA, ipsi addendo dimidium anguli Z.

4. Observari etiam debet, quòd distantia quæ invenitur per analysim trianguli plani, semper sit via brevissima, seu linea recta connectens ea loca, quorum distantia investigatur. *Ex. gr.* Si exploretur in Sphæra terrestri distantia inter C & B, non invenietur arcus CB, qui ob terræ rotunditatem est vera distantia; sed illius subtensa, seu recta CB, quæ cùm minor sit arcu, aliquantulum deficit à distantia quæsita. Quia verò defectus ille in distantiâ non valdè magnâ est insensibilis, ideò meritò negligitur. Quòd si contingat ut proponatur distantia maxima, eique requiratur par accuratio; sic facilè obtinebitur. 1.° Quæratur (*juxta hoc, vel sequens*

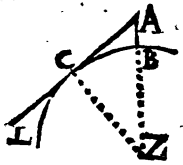
Probl.) angulus Z, seu arcus CB, qui sit vg. 1 gr. 30'. 2.° Posito quòd ZB Terræ semidiameter contineat qualescunque partes 10000000; resolvendo triangulum ZBC, invenietur, subtensa CB esse ad latus ZB, ut 261792 ad 10000000; adeoque recta CB est ad totam diametrum ut 261792 ad 20000000. 3.° Quia diameter circuli (vide P. Tacquet *scholio in Prop. 6. selectarum ex Archimede*) est ad circumferentiam, ut 7 ad 22, vel exactiùs ut 113 ad 355, ut habeatur proportio inter diametrum sic divisam & totam circumferentiam, fiat ut 113, ad 20000000; sic 355 ad aliud, & invenietur tota circumferentia ad diametrum, ut 62831859 ad 20000000. 4.° Fiat ut 360 gr. ad 1 gr. 30', seu ut 21600 ad 90', ita circumferentia 62831859 ad aliud, & quartus terminus 261799 indicabit rationem arcûs CB ad diametrum 20000000. Sed ex dictis subtensa CB est ad diametrum, ut 261792 ad 20000000; ergo subtensa CB, est ad arcum CB, ut 261792 ad 261799. 5.° Ponamus quòd subtensa CB inventa fuerit passuum 96544 & pedum $2\frac{1}{2}$, id est (supponendo pedem dividi, uti communiùs dividi solet, in 12 digitos) digitorum 5792670; Dicendo, ut subtensa CB 261792 ad digitos 5792670: sic arcus CB 261799 ad aliud; invenietur arcus CB continere digitos 5792823 seu passus 96547 $\frac{3}{60}$; adeoque differentia inter arcum & subtensam est passus 2. pedes $2\frac{3}{4}$.

Hinc confirmatur quòd diximus, viz: excessum arcûs supra subtensam sine dispendio negligi in mediocri intervallo, cùm in distantia Mill. Bon: 96 $\frac{1}{2}$ seu leucarum 32 circiter, sit tantilla differentia.

Corollaria.

1. Notâ terræ semidiametro ZC seu ZB, & altitudine oculi AB, sic invenitur amplitudo horizontis Physici. *Vide definitionem 4. cap. 5. P. 2.*

In triangulo ACZ angulus C est rectus, latus CZ terræ semidiameter, & latus AZ habetur addendo oculi altitudinem



AB semidiametro terrestri ZB, & his tribus invenitur angulus Z, adeoque & arcus CB, qui est dimidia amplitudo horizontis Physici, & ad milliaria reducetur modo statim explicando.

2. Eodem modo determinatur distantia (CB) ex qua quævis altitudo nota (AB) potest conspici: & è converso, notâ distantiâ invenitur altitudo.

Nam si detur altitudo AB, nota erunt in triangulo CAZ, latera CZ, AZ cum angulo recto C; ex quibus invenitur angulus Z, seu arcus CB distantia maxima ex qua punctum A potest conspici.

E converso, si cognoscatur distantia seu arcus CB, notus erit angulus Z, & latus CZ cum angulo C recto. Ex his tribus quæratur AZ ut subtractâ BZ terræ semidiametro, inveniatur AB altitudo quæsita.

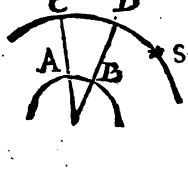
3. Ex eadem figura colligitur quòd in distantia magna non rectè exploretur angulus distantiæ CAB quadrante & perpendiculo, nisi correctio aliqua adhibeatur. Nam qui ex C collimaret in A, tendente perpendiculo ad centrum Z per rectam CZ, inveniret angulum LCZ, qui (*per* 32. *l.* 1.) æquatur angulis Z & CAB quæsito. Subtrahi igitur debet ex angulo invento LCZ angulus Z, qui (*ex dicendis*) facilè innotescet notâ distantiâ CB.

4. Ex terræ diametro notâ deducitur ejus circumferentia. Securior tamen est terrestris circumferentiæ investigatio immediata, quæ sequitur.

PROBLEMA II.

Terræ circumferentiam invenire.

PRIMUS MODUS.

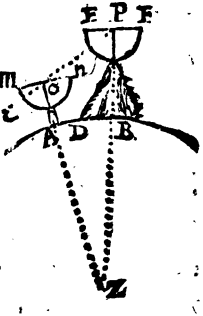


Eligantur sub eodem meridiano duo loca A & B, quorum distantia diligenter exploretur. Sit loci A, Zenith C; loci B, Zenith D. Jam, si in utroque loco observetur alicujus stellæ S altitudo meridiana, nota erunt altitudinum complementa

SD, SC, eorumque differentia CD; adeoque & arcus terrestris AB, similis arcui CD. Fiat igitur ut arcus AB, ad locorum distantiam; sic tota terræ circumferentia, seu gradus 360. ad aliud; & ostendet quartus terminus quot sint milliaria in toto terræ ambitu.

SECUNDUS MODUS.

Sit P locus in monte à longè conspicuus, & A turris in planitie milliaribus aliquot distante. Montis & turris altitudines perpendiculares PB, OA concurrentes in terræ centro Z, inter quas sit distantia horizontalis AB. His positis,



1.° Quadrante vel semicirculo ita collocato in P ut ejus latus EF sit horizontale, adeoque rectum lineæ PB, metire inclinationis angulum CPB, quem efficit radius visualis PC radens turrim AO; dein Telescopio observetur illud in turri punctum O, quod stringit radius visualis PC.

2.° In parte turris observatâ O, idem instrumentum ita statue, ut latus MN sit horizontale, & rectum lineæ OA. Deinde collimans in P, ubi prima facta est observatio, nota exactè elevationis angulum AOP, cujus supra rectum excessus NOP (*per.* 15. *l.* 1.) æquatur opposito ad verticem MOC.

3.° In triangulo POZ, inventis jam angulis OPZ, POZ, notus erit (*per.* 4. *Corol. p.* 32. *l.* 1.) tertius Z: ergo notum etiam erit quot gradus vel minuta contineat arcus AB. Si igitur (*per Probl.* 3. *c.* 5.) quæratur AD, & (*per* 1 *Corol. Probl.* 7. *c.* 5.) DB, ut cognoscatur in milliaribus vel passibus tota distantia AB; invenientur (*ut in* 1. *modo*) milliaria vel passus quos continet tota terrestris Sphæræ circumferentia.

Scholium.

Ex hac operatione sæpiùs peracta, tribuit Ricciolus terræ circumferentiæ Milliaria Bononiensia 23170, & Passus 6801 semidiametro terræ Mill: Bon. 3689. Passus 598; Uni Gradui

circuli maximi Mill. 64. Passus 363: Areæ circuli terrestris maximi Mill: quadrata 42745247: superficiei terrestri Mill: quadrata 170980988: soliditati terræ Mill. cubica 210283703785.

Mathematici Parisienses apud Ozanam Tom. 5. pag. 149 assignant uni gradui Passus Bon: 58481, seu Mill. Bon. $58\frac{1}{2}$ circiter. Unde ex illorum calculo continebit terræ ambitus Mill: Bon. 21053, & Passus 160.

Has terræ dimensiones mensuris Bononiensibus expressas ad Patriæ suæ milliaria quivis facilè reducat, modò determinetur illa quæ est inter aliquas mensuras notas, pedes vg. proportio. Cùm enim Milliare Bon: contineat 1000 passus, & passus singuli 5 pedes; *per Auream Regulam*, notâ pedum proportione, constabit passuum & milliarium proportio; modò sciatur quot pedes vel passus ad alicujus regionis milliare requirantur.

Ut faciliùs peragatur hæc operatio, pedum celebriorum proportionem hîc subjicio. Quarum partium continet *Pes Bononiensis* 1686, earundem continet *Romanus antiquus* 1331. *Parisiensis* 1440. *Londinensis* 1350. *Leodiensis* 1320 *Rhynlandicus seu Leydensis* 1390. *Antuerpiensis* 1277. *Bavarus* 1284. *Amstelodamensis* 1256. *Lovaniensis* 1263. *Norimbergensis* 1353. *Venetus* 1557.

Hoc semel invento, quot alicujus loci milliaria correspondeant uni gradui maximi terræ circuli, *per Regulam Auream* invenientur gradus correspondentes cuicunque intervallo noto; & è converso.

Ex: gr: Inquiratur arcus correspondens milliaribus 342. Fiat ut mill: $64\frac{363}{1000}$, seu passus 64363, ad 1 gradum seu 60 min: sic mill: 342 seu passus 342000, ad aliud; & quartus terminus 5 gr. 18' erit quantitas arcûs quæsiti.

PROBLEMA III.

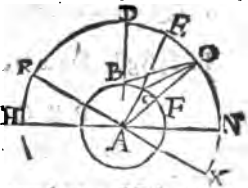
Lunæ à Terra distantiam metiri.

Eligantur duo loca B & C sub eodem Meridiano HDX, in quo sit Lunæ locus O. Loci B, Horizon sit HN, Zenith D.

Loci

Loci C, Horizon RX, Zenith E. His positis;

1.° Duo mensores eâdem nocte conflictâ, quando Luna appellit ad Meridianum, metiantur angulos distantiæ Lunæ à suo quisque Zenith, *viz:* angulos OBD, OCE.



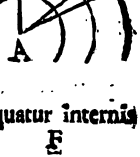
2.° Inquiratur distantia locorum B & C, ut sciatur arcus BC, seu angulus BAC; & nota erit (*per. 4. Corol. p. 32. l. 1.*) summa angulorum ABC, ACB; qui cùm (*per. 5. l. 1.*) sint æquales, singuli cognoscentur. Ex omnibus angulis & BA terræ semidiametro, quæratur subtenta BC.

3.° Angulis ABC, OBD jam notis, cognoscetur intermedius OBC eorum complementum ad duos rectos: & noto angulo A B, scietur BCE ejus complementum ad duos rectos; cui si addideris OCE, notus erit BCO. Ergo in triangulo OCB ex angulis BCO, CBO, & latere BC, invenientur BO Lunæ distantia à loco B, & CO ejusdem distantia à loco C.

4.° In triangulo ABO, ex notis lateribus AB, BO, & angulo ABO conflato ex notis ABC, CBO, invenitur AO distantia Lunæ à centro terræ; & (subtractâ AF) FO, distantia Lunæ ab F, terræ puncto sibi directè subjecto.

Notandum.

Lunæ aliorumque Planetarum variatur sæpe à terrâ distantia. Hoc colligitur 1.° ex eo quòd (etiam abstrahendo à refractione) nunc sub majori, nunc sub minori angulo conspiciantur. 2.° Ex varietate Parallaxium seu angulorum AOB, ACB, quos in centro Planetæ efficiunt duo radii, quorum unus ducitur ex A centro terræ, alter ab oculo B in terræ superficie posito.



Quòd autem, dum Planeta vicinior est in O, major sit parallaxis BOA, quàm BCA parallaxis ejusdem dum remotior est in C, patet; nam (*per. 32. l. 1.*) angulus BOA æquatur internis BCA, OAC; adeoque excedit BCA.

Ricciolus ex obſervationibus plurimis relatis *libro 4. Almageſti. pag.* 225. deducit Lunæ diſtantias ſequentes.

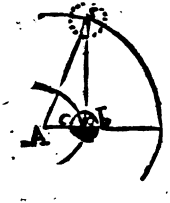
In Quadraturis Lunæ	Apogææ 66 -- 42′.	}	Terræ
	Perigææ 51 -- 20.	}	
In Syſygiis Lunæ	Apogææ 64 -- 15	}	Semidiametros.
	Perigææ 53 -- 45	}	

PROBLEMA IV.

Solis à Terra diſtantiam inveſtigare.

Sit in Sphæra terreſtri oculi locus A, Lunæ centrum o, ſolis centrum S.

1.o Quando Luna dichotoma eſt, ſeu apparet dimidiâ ſui parte illuminata, planum *b o c* ſeparans partem lucidam ab obſcura, ſi produceretur, tranſiret per oculum exiſtentem in A; adeoque radius viſualis A *o*, tranſiens per Lunæ centrum, eſt in illo plano. Patet; quia aliter una ſemiſſis hæmiſphærii illuminati non occultaret aliam.



2.o Recta S *o*, Solis & Lunæ dichotomæ centra connectens, eſt perpendicularis plano *b o c*; adeoque (*per. 3. def. l.* 11.) etiam radio viſuali A *o*, exiſtenti in illo plano. Patet; nam hæmiſphærium illuminatum, adeoque & ipſius baſis, ſeu planum *b o c*, ſoli directè obvertitur. His præmiſſis,

In triangulo AS*o* latus A*o* eſt Lunæ à terrâ diſtantia (*per præced*) nota; angulus S*o*A eſt rectus, & angulus *o*AS ad dichotomiæ momentum debet inſtrumento exquiſito explorari. Ex his tribus invenietur SA, Solis diſtantia à terra.

Hanc aliaſque diſtantiæ ſolaris inveſtigandæ methodos fuſiùs explicatas vide apud Ricciolum *in Almageſto*, *l.* 3. *c.* 7. cum ipſius, & præcipuorum Aſtronomorum de Solis diſtantia judiciis. Tandem (*Aſtronomia Reformata l.* 1. *c.* 13.) conſtituit Solis diſtantiam maximam terræ ſemidiametros 7427: minimam 7173.

Scholium.

Reliquorum Planetarum & Fixarum diſtantiam inveſti-

gare operosum est, & multorum notitiam prærequirit, quorum explicatio alterius est loci. Subjiciam proinde tabulam distantiarum quas ex suis observationibus infert Riccioli (*Astron. Reform. l.* 10. *c.* 15.)

		maxima.	*minima.*	
A Terra distantia:	Mercurii	10435.	4948.	Terræ Semidiametris
	Veneris	12754.	1946.	
	Martis	19576.	2670.	
	Jovis	45669.	28864.	
	Saturni	81105.	58207.	

Si Fixæ haberent parallaxim saltem 2 secundorum, inde deduceretur earum distantia à Terra major Terræ Semidiametris 100000. Cùm igitur parallaxis sit ita exigua, ut nullâ Astronomorum industriâ unquam potuerit observari, certum est earum distantiam esse multò majorem. Quanta verò sit, nequit certò determinari.

Fixarum distantia duplicata dat diametrum Firmamenti majorem Terræ semid. 200000, adeoque continentem ad minimum Passus 737919600000. Unde circumferentia Firmamenti erit Passuum plusquàm 2318244525762. Cùm igitur Firmamentum cum omnibus stellis fixis integram revolutionem peragat 24 horis, hinc colligitur stellam sub Æquatore positam conficere minutis singulis Passus 1609892031 $\frac{5}{9}$. qui motus si conferatur cum circumferentia Terræ continente Passus 23170680, constabit stellas sub Æquatore sitas conficere minutis singulis plusquàm sexagies nonies totius Terræ ambitum. Quod tametsi mirum videri possit, est tamen infra veritatem.

PROBLEMA V.

Notâ cujuscunque sideris distantiâ, & diametro apparenti, ejus veram diametrum invenire.

Sit alicujus astri distantia AB, diameter apparens angulus CAD factus à radiis visualibus AC, AD, qui ducti ex oculi loco A sideris sphæram tangunt in C & D: sint etiam ejusdem sphæræ radii BC, BD, & diameter vera XZ.

In triangulis ABC, ABD, latera BC, BD (*per.* 18. *def. l.* 1.)

& latera AC, AD (*per.* 2. *Corol.* 36. *l.* 3. sunt æqualia, & latus AB commune: ergo (*per.* 8. *l.* 1.) anguli BAC, BAD sunt æquales, adeoque BAC est semissis anguli CAD (*ex suppos.*) noti. Ergo in triangulo ABC noti sunt anguli BAC, & BCA (*per.* 18. *l.* 3.) rectus, cum latere AB; ex quibus invenitur BC, seu BX semidiameter, quæ duplicata dat diametrum veram XZ.

Scholium.

Modus inveniendi siderum diametros apparentes, quem brevitatis causâ prætermitto, videri potest apud Dechales *Astronomiæ lib.* 4. *Prop.* 27. &c. Ex illis autem quas assignat Ricciolus, & distantiis suprà positis, deducuntur eorum veræ diametri ad terræ diametros se habere,

Solis. ut 33 ad 1. *Lunæ.* ut $26\frac{1}{2}$ ad 100. *Mercurii* ut 26 ad 100. *Veneris.* $1\frac{1}{10}$ ad 1. *Martis* ut 100 ad 59. *Jovis* ut 9 ad 2. *Saturni* ut $4\frac{3}{10}$ ad 1. *Sirii* $4\frac{9}{25}$ ad 1. *Capellæ.* ut $3\frac{21}{100}$ ad 1.

Quia sphærarum proportio triplicata est proportionis diametrorum, (*vide apud P. Tacquet. Corol. p.* 18. *lib.* 12. *Euclidis.*) infertur Sphæram Terrestrem esse ad Solem, ut 1 ad 35937: ad Lunam, ut 51 ad 1: ad Mercurium, ut 57 ferè ad 1: ad Venerem, ut 100 ad 133: ad Martem, ut 48 ad 10: ad Jovem, ut 8 ad 729: ad Saturnum, ut 1 ad 795: ad Sirium, ut 1 ad 83 ferè: ad Firmamentum denique, ut 1 ad 1000000000000000.

Ex dictis pronum erit observare quàm exilis sit Terrarum orbis, si cum mundi spectabilis amplitudine conferatur. Quid porrò erit si comparetur cum cælo empyreo, fortunatâ Beatorum sede? Si hæc eo, quo par est animo, expenderit pius Lector, sordebit profectò Terra, dum cælum & cælestia contemplatur.

PARS SECUNDA.

DE TRIGONOMETRIA SPHÆRICA.

Artem hanc in quinque capita distribuemus. *Primò* præmittemus nonnulla ad trianguli sphærici proprietates intelligendas necessaria. *Secundò* trianguli sphærici rectanguli, *Tertiò* obliquanguli solvendi modum indicabimus. *Quartò* pro determinanda laterum & angulorum specie, regulas aliquas præscribemus. *Quintò* proponemus Problemata aliquot nobiliora, quorum solutione præcedentium Capitum doctrina illustrabitur.

CAPUT I.

Præmittuntur aliqua ad Trigonometriam Sphæricam.

DEFINITIONES.

1.ª *Sphæra* est figura solida, unicâ superficie comprehensa, ad quam omnes rectæ ductæ ab aliquo intra sphæram puncto (quod *Centrum* dicitur) sunt æquales.

2.ª *Circulus maximus* est quô major non potest in superficie sphæræ describi.

3.ª *Polus* circuli est punctum in superficie sphæræ, ab istius circuli circumferentia distans undique æqualiter.

4.ª *Angulus Sphæricus* (de quo hîc agimus) fit à duobus circulorum maximorum arcubus; & tot continet gradus, quot continet angulus quem faciunt circulorum plana.

Præter pleraque in Parte præcedenti recensita, ex Euclide ad Trigonometriam Sphæricam requiruntur ex Libro 1. *Prop.* 20. 47. *cum* 4. & 5. *Corol. Ex Libro* 3. *Prop.* 7. 9. 10.

13. 29. *cum* 1. *Corol. Ex Libro* 5. *Prop.* 22. 23. *Ex Libro* 11. *Prop.* 9. 15.

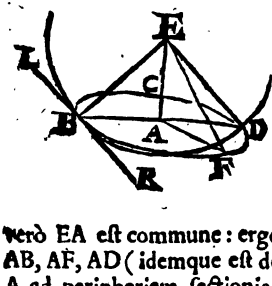
Nota. Quòd tametsi quæ capite hoc continentur ad triangulí sphærici proprietates penitiùs inspiciendas multùm juvent, non tamen omnia ad trianguli resolutionem intelligendam sint absolutè necessaria. Tyroni proinde suaserim, ut Definitionibus & prioribus 7 Propositionibus contentus, Caput secundum auspicetur; ad reliqua, ubi posteà citantur, commodiùs recurret.

PROPOSITIO I.

Si sphæra plano quomodocunque secetur, sectio communis est circulus.

Demonstr. 1.° Si planum transeat per centrum, patet, nam omnes lineæ à centro sphæræ ad superficiem ductæ sunt æquales.

2.° Si non transeat per centrum, ad planum illud à centro ducatur perpendicularis EA, & ex A ducantur ad sectionis perimetrum BCDF quotcunque rectæ AB, AF, AD, quibus occurrant radii sphæræ EB, EF, ED. In triangulis EAB, EAF, EAD, anguli ad A (*per* 3. *def. l.* 11.) sunt recti, & hypotenusæ EB, EF, ED (*per* 1. *def. h*) æquales: latus verò EA est commune: ergo (*per* 4. *Corol. p.* 47. *l.* 1.) latera AB, AF, AD (idemque est de omnibus quæ duci possunt ab A ad peripheriam sectionis) sunt æqualia. Ergo (*per* 18. *def. l.* 1.) sectio communis est circulus, cujus circumferentia est BCDF, & centrum A.



Corollaria.

1. Circulus maximus est qui transit per centrum sphæræ. Nam radii talis circuli sunt, ut patet, iidem cum radiis sphæræ, & centrum idem: sed radii cujuscunque circuli non transeuntis per centrum sphæræ sunt minores radio sphæræ, ut patet in circulo BCDF, cujus radii sunt (*per* 18. *l.* 1.) minores radiis sphæræ oppositis angulo recto A.

2. Circulus duci potest per quæcunque tria puncta in superficie sphæræ. Nam per illa puncta duci potest planum, cujus cum sphærâ sectio communis est circulus.

3. Circuli per illa tria puncta descripti circumferentia tota est in superficie sphæræ. Nam planum per illa tria puncta ductum facit circulum, cujus circumferentia est tota in sphæræ superficie, ut patet. Sed per eadem tria puncta non potest duci nisi unus circulus; aliter enim duo circuli se secarent aut tangerent in tribus punctis, quod est (*per.* 10. & 13. *l.* 3.) impossibile: ergo.

4. Recta LR tangens aliquem in sphæra circulum BCDF, tangit ipsam sphæram: nam cùm circuli BCDF circumferentia sit tota in sphæræ superficie, & LR in eodem plano cum circulo BCDF, patet quòd LR non possit sphæram secare, quin secet circulum BCDF; adeoque non sit illius tangens.

PROPOSITIO II.

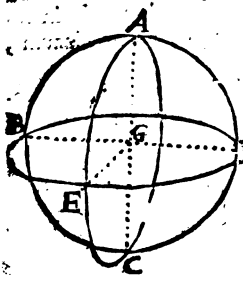
Arcus insistens arcui facit angulos duobus rectis pares, & arcus se intersecantes faciunt angulos ad verticem oppositos æquales.

Demonstr. Plana (*per 6. def. l.* 11.) faciunt eosdem angulos quos lineæ ductæ perpendiculariter ad sectionem communem. Sed lineæ istæ, quando planum insistit plano, adeoque linea lineæ, faciunt (*per 13. l.* 1.) angulos æquales duobus rectis: & quando plana, adeoque lineæ in planis ductæ, se interfecant, faciunt (*per* 15. *l.* 1.) angulos æquales ad verticem oppositos: ergo plana faciunt dictos angulos. Sed (*per 4. def. h*) arcus faciunt eosdem angulos, quos circulorum plana: ergo.

PROPOSITIO III.

Mensura cujusvis anguli sphærici (BAE) *est circuli maximi arcus* (BE) *interceptus inter* AB, AE) *latera ipsius anguli, & utrinque distans* 90 *gradibus ab* (A) *anguli vertice.*

Demonstr. Quia AB, AE sunt arcus 90 graduum, anguli



AGB, AGE sunt recti: ergo lineæ EG, BG sunt perpendiculares ad planorum sectionem AC: ergo (*per def. 6. l. 11.*) angulus BGE est angulus inclinationis planorum; adeoque (*per 4. def. h*) æquatur angulo sphærico BAE: sed arcus BE est mensura anguli BGE: ergo & anguli BAE.

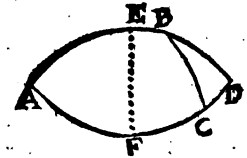
PROPOSITIO IV.

In Sphæra circuli maximi se mutuò bisecant.

Demonstr. Centrum sphæræ (*per 1. Corol. 1. h*) est centrum circulorum maximorum: ergo omnes circuli maximi habent idem centrum: ergo communis illorum sectio transit per centrum commune; adeoque est (*per 20. def. l. 1.*) communis eorum diameter, eósque bisecat in duos semicirculos.

Corollaria.

1. Si trianguli sphærici ABC duo quævis latera AB, AC, quæ in uno sectionis extremo faciunt angulum A, producantur donec fiant semicirculi, concurrant in altero extremo D.

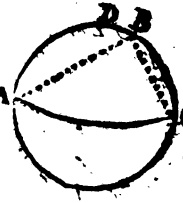


2. Dum trianguli ABC latera sic producta concurrunt, fit novum triangulum, in quo 1.o habetur angulus D æqualis angulo A, nam idem arcus EF est (*per 3. h*) utriusque mensura. 2.o Latus BC est utrique triangulo commune. 3.o Reliqua latera BD, CD sunt laterum AB, AC complementa ad semicirculum. 4.o Anguli CBD, BCD sunt angulorum ABC, ACB complementa ad duos rectos. Unde 5.o Hoc ipso quòd cognoscantur aliqui anguli vel latera in uno triangulo, cognoscuntur correspondentes anguli & latera in alio. 6.o Iidem sunt sinus correspondentium tum laterum, tum angulorum in utroque triangulo.

PRO-

PROPOSITIO V.

Polus circuli maximi distat undique 90 gradibus ab ejus circumferentia.



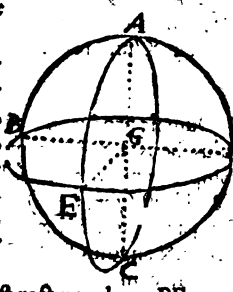
Demonstr. Si neges: Sit circuli AC Polus B, distans minùs quadrante à C. Quia ADBC est (*per 4. h.*) semicirculus, arcus ADB major est arcu BC, adeoque subtensa AB major subtensâ BC: ergo B minùs distat à C quàm ab A: ergo (*per 3. def. h.*) B non est polus.

Corollarium.

Eodem modo ostenditur circulum quemvis unicum habere polum in eodem hæmisphærio. Si possint esse duo, sint D & B. Per D & B ducatur arcus ADBC. Patet puncta D & B non posse ambo distare 90 gradibus ab A & C.

PROPOSITIO VI.

Circulus maximus (ABCD) *transiens per* (A) *polum alterius circuli maximi* (BED) *illi insistit ad angulos rectos: & insistens ad angulos rectos transit per polum.*



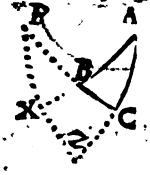
Demonstr. I. Pars. Sumatur arcus BE 90 graduum, & ex E ducatur arcus EA. In triangulo ABE, arcus BE (*ex suppos.*) est quadrans, & BA (*per 5. h.*) est etiam quadrans: ergo (*per 3. h.*) arcus EA est mensura anguli ABE. Sed EA (*per 5. h.*) est quadrans: ergo (*per 3. h.*) angulus ABE est rectus: ergo (*per 4. def. h.*) planum AB est rectum plano BE.

2. Pars. Si insistens ad angulos rectos non transeat per polum, inclinatus in unam vel aliam partem transibit per polum: ergo (*per I. partem*) sic inclinatus insistet ad angulos rectos, quod est absurdum.

PROPOSITIO VII.

Si trianguli rectanguli sphærici (ABC) *cujus singuli arcus sunt quadrante minores, latera omnia producantur donec fiunt quadrantes* (ACZ, ABX, CBP) *& jungantur puncta* P *&* Z *arcu circuli maximi* (PZ); *hic arcus* (PZ) 1° *erit quadrans*, 2° *transibit per* (X) *extremum quadrantis intermedii* (ABX) 3° *faciet* (ad X) *angulos rectos.*

Demonstr. 1. Pars. Quia angulus aliquis C est (*ex suppos.*) rectus, arcus CBP (*per 6. h.*) transit per polum circuli ACZ : cùm igitur CBP sit (*ex constr.*) quadrans, punctum P (*per 5. h.*) est ille polus: ergo (*per eandem*) arcus ZP est quadrans.



2. *Pars.* Quia ZP transit per P polum circuli ACZ, angulus Z (*per 6. h.*) est rectus : ergo (*per eandem*) ACZ transit per polum circuli ZP; & quia ACZ (*ex constr.*) est quadrans, punctum A (*per 5. h.*) est ille polus : ergo cùm ABX sit (*ex constr.*) quadrans, præcisè pertingit (*per 5. h.*) à polo A ad arcum ZP, qui proinde transit per X.

3. *Pars.* Quia arcus XBA transit per A polum arcûs ZP, illi insistit (*per 6. h.*) ad angulos rectos; adeoque anguli ad X sunt recti.

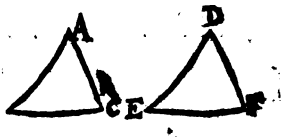
Corollarium.

Dum sic producuntur latera, fit novum triangulum BPX, cujus latus BX est complementum arcûs AB; & latus BP complementum arcûs CB; & latus PX complementum arcûs XZ, id est (*per. 3. h.*) anguli A. Angulus verò PXB est rectus; angulus PBX æqualis (*per. 2. h.*) oppositò ad verticem ABC; & angulus BPX (*per. 3. h.*) mensuratur ab arcu CZ, qui est complementum arcûs AC. Unde hoc ipso quòd noti sint aliqui anguli vel latera trianguli ABC, cognoscuntur correspondentes anguli & latera in triangulo BPX: & è contrà.

PROPOSITIO VIII.

Si in duobus triangulis (ABC, DEF) *æqualia sint duo*

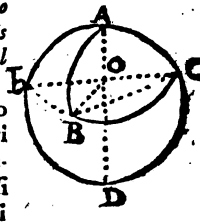
latera (AB, DE, & AC, DF) & *anguli contenti* (A & D) *æquales*; *vel si basis* (BC) *æquetur basi* (EF,) & *anguli adjacentes basi* (B & E, C & F) *sint æquales*; *tota triangula erunt æqualia.*



Hæc propositio correspondet quartæ l. 1. ejusque scholio, & eodem modo demonstretur; nam si unum triangulum alteri imponas, perfectè congruent.

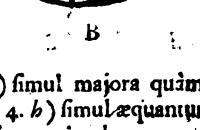
PROPOSITIO IX.

Cujusvis trianguli sphærici (ABC) *duo quævis latera* (AB, AC) *sunt simul sumpta majora tertio* (BC) *item quodvis latus* (BC) *est minus semicirculo, & omnia tria latera simul sumpta minora circulo.*



Demonstr. I. Pars. Perfecto circulo AC, acceptoque arcu A*b* æquali lateri AB, ducantur chordæ BC, & *b*OC secans diametrum AD alicubi in O. Jam si semicirculus A*b*D concipiatur moveri circa communem diametrum DA donec coincidat cum plano arcûs AB, quia A*b* AB sunt (*ex constr.*) æquales arcus æqualium circulorum, punctum *b* incidet in B, & recta *b*O congruet rectæ BO. Sed (*per* 20. *l.* 1.) BO, OC, id est subtensa *b*C major est subtensâ BC: ergo (*per.* 26. *l.* 3.) arcus *b*AC seu latera AB, AC, excedunt arcum seu latus BC.

2. *Pars.* Quia BC, AC tantùm concurrunt in uno extremo C, si producantur, (*per* 1. *Corol.* 4. *h*) concurrent in altero sectionis extremo D, & erit CBD semicirculus: ergo arcus BC est minor semicirculo.



3. *Pars* DB, DA sunt (*per I. partem*) simul majora quàm AB. Sed CA, CB cum DA, DB, (*per* 4. *h*) simul æquantur circulo: ergo CA, CB cum AB sunt minora circulo.

PROPOSITIO X.

Cujusvis trianguli sphærici (ABC) *quivis angulus* (C) *minor est duobus rectis : omnes anguli simùl sumpti sint minores sex rectis, & majores duobus rectis.*

Demonstr. I. Pars. Producantur latus BC. Anguli X & C (*per* 2. *h*) æquantur duobus rectis : ergo angulus C est minor duobus rectis.

2. *Pars* patet ex prima.

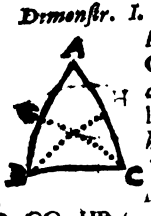
3. *Pars* 1° Si anguli X & B sint æquales, sicut X cum C (*per* 2. *h*) ita B cum C faciet duos rectos : ergo anguli B, C, A, sunt majores duobus rectis.

2.° Si X sit minor quàm B, quia X & C faciunt duos rectos ; B & C, adeoque à fortiori B, A & C, erunt majores duobus rectis.

3.° Sit X major quàm B. Tres angulos A, B, C excedere duos rectos postea commodiùs demonstrabimus. *Vide Corol. Prop.* 13. *quæ ab hac non dependet.*

PROPOSITIO XI.

Si in triangulo sphærico (ABC) *duo latera* (AB, AC) *sint æqualia, anguli iis oppositi* (B & C) *sunt æquales : & è converso.*



Demonstr. I. Pars. Ex AB, AC aufer arcus æquales AG, AH, & describe arcus GC, BH. Trianguli GAC, HAB habent latera AG, AH (*ex constr.*) & latera AB, AC (*ex suppos.*) æqualia, & angulum A communem : ergo (*per* 8. *h*) æquantur bases GC, HB, & anguli AGC, AHB ; adeoque & horum complementa BGC, CHC sunt æqualia. Sed etiam latera GB, HC & GC, HB sunt æqualia : ergo anguli GBC, HCB, id est B & C sunt (*per* 8. *h*) æquales.

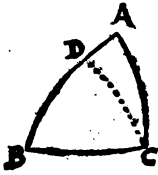
2. *Pars.* Si latera AB, AC non sint æqualia, sit AB majus. Ex AB aufer BD æquale AC, & duc arcum DC.

In triangulis BDC, BAC latus BC est commune, latus BD (*ex constr.*) æquatur lateri AC, & angulus contentus B (*ex suppos.*) angulo contento C: ergo (*per* 8. *h*) triangula BDC, BAC sunt æqualia, pars & totum.

PROPOSITIO XII.

In triangulo quovis sphærico (ABC) *majori angulo* (C) *opponitur majus latus* (AB: *&* è *converso.*

Demonst. I. Pars. Fiat angulus DCB æqualis angulo B, & erunt (*per* 11. *h*) CD, BD æqualia. Sed (*per* 9. *h*) CD, DA simul sunt majora quàm AC: ergo DB cum DA, seu AB majus est quàm AC.

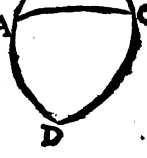


2. Pars. Si anguli B & C essent æquales, latera AB, AC essent æqualia (*per* 11. *h.*) Et si C esset minor quam B, arcus AB (*per I. partem*) foret minor quàm AC: ergo si AB sit major quàm AC, angulus C est major quam B.

PROPOSITIO XIII.

Si trianguli sphærici (ABC) *duo quævis latera* (AB, AC) *simul sumpta sint æqualia semicirculo, anguli* (B & C) *ad basim erunt simul æquales duobus rectis; si majora sint, anguli sunt majores; si minora, anguli sunt minores duobus rectis: & è converso.*

Demonstr. I. Pars. Producantur BA, BC, donec concurrant in D. Jam quia BA, AC [*ex suppos.*] & BA, AD (*per* 4. *h*) æquantur semicirculo; AC, AD sunt æqualia: ergo (*per* 11. *h*) angulus ACD æquatur angulo D, id est (*per* 2. *Corol. p.* 4. *h*) angulo B. Sed anguli ACB cum ACD (*per* 2. *h*) æquant duos rectos: ergo ACB cum B facit duos rectos.



2. Pars. Si B, id est (*per* 2. *Corol. p.* 4. *h*) D cum ACB æquat duos rectos, quia ACD cum eo-

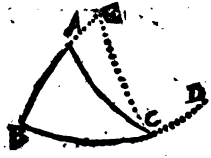
dem ACB (*per* 2. *h*) idem facit; anguli D & ACD sunt æquales: ergo (*per* 11. *h*) æqualia sunt latera AC, AD. Sed AB, AD sunt (*per* 2. *Corol. p.* 4. *h*) semicirculus: ergo AB, AC æquantur semicirculo.

3. *Pars.* Si AB, AC sint semicirculo majora, AC est (ut patet ex dictis) majus quàm AD: ergo angulus D, adeoque & B, est (*per* 12. *h*) major angulo ACD. Sed ACD cum ACB (*per* 2. *h*) facit duos rectos: ergo B cum ACB facit plusquàm duos rectos.

Conversa hujus, sicut & tertia pars cum suâ conversâ similî ratiocinio demonstratur.

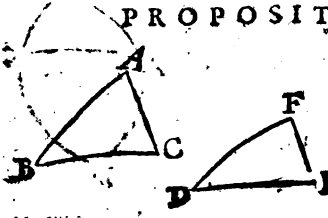
Corollarium.

Si angulus externus ACD *sit major interno opposito* B, *anguli interni* A, B, C *sunt majores duobus rectis.*



Fiat angulus ECD æqualis angulo B, & producatur BA donec concurrat cum CE. Jam angulus ECB cum ECD, id est (*ex constr.*) cum B adæquat duos rectos: ergo (*per* 13. *h*) latera BE, CE æquantur semicirculo: ergo latera AE, EC sunt simul semicirculo minora: ergo (*per* 13. *h*) anguli EAC, ECA sunt minores duobus rectis: ergo BAC, qui cum EAC (*per* 2. *h*) æquat duos rectos, est major quàm ECA. Si ergo angulo BAC addas angulum B, & angulo ACE angulum ECD æqualem ipsi B, & utrinque adjicias angulum ACB: erunt tres anguli BAC, ACB, CBA majores tribus ACE, ACB, ECD, id est (*per* 2. *h*) duobus rectis.

PROPOSITIO XIV.

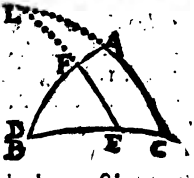


Si duo triangula sphærica (ABC, DEF) *habeant latera* (AB, AC, BC) *singula æqualia singulis* (FD, FE, DE) *anguli æqualibus lateribus oppositi sunt æquales: & è converso*

Hujus Propositionis prima pars correspondet octavæ l. 1. Euclidis; & quemadmodum illa *per* 4. & 5. *l.* 1. *Euclidis* demonstrata est, ita & hæc eodem modo potest demonstrari *per* 8. & 11. *h.*

Demonstr. 2. *Pars.* Quòd triangula sibi mutuò æquiangula, sint etiam æquilatera si habeant latus aliquod commune vel æquale, patet *ex* 8. *h.* Donec igitur demonstretur contrarium, fingamus omnia latera esse inæqualia. Et

1.o Posito angulo D super æqualem B, latera DE, DF minora sint lateribus BC, BA. Producantur EF, CA donec concurrunt in puncto L. Anguli FED, FEC (*per* 2. *h*) æquantur duobus rectis. Sed FED & ACB (*ex suppos.*) sunt æquales: ergo FEC, ACB æquantur duobus rectis: ergo trianguli LEC latera LE, LC (*per* 13. *h*) æquantur semicirculo. Similiter anguli EFA, EFD (*per* 2. *h*) æquant duos rectos. Sed EFD, CAB (*ex suppos.*) sunt æquales: ergo EFA, CAB æquantur duobus rectis: ergo (*per* 2. *h*) eorum complementa LFA, LAF æquantur duobus rectis: ergo (*per* 13. *h*) latera LF, LA æquantur semicirculo, quod est absurdum, quia arcus LC, LE jam ostensi sunt semicirculo æquales.



2.o Si latera DE, DF ponantur singula majora lateribus BC, BA; eadem fiet demonstratio.

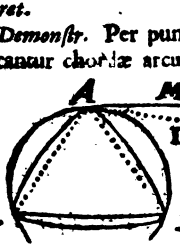
3.o Si unum DF ponatur majus quàm BA, & aliud DE minus quàm BC; ostendetur, ut priùs, arcus LE, LC, item arcus LA, LF esse æquales semicirculo: ergo latera AC, FE vel sunt semicirculi, vel aliquod eorum majus semicirculo; utrumque verò est (*per* 9. *h*) impossibile: ergo latera singula trianguli DEF æquantur singulis trianguli BCA.

PROPOSITIO XV.

Si in triangulo spherico Isosceli (ADC) *ducatur chorda basis* (CD) & *per* (A) *verticem anguli oppositi transeat*

recta (MA) *chordæ* (CD) *parallela, illa recta* (MA) *tanget sphæram in qua est triangulum* (ACD) *ita ut in eam non intret.*

Demonstr. Per puncta A, D, C, describatur circulus & ducantur chordæ arcuum AC, AD, quæ (*per 26. l. 3.*) sunt



æquales. Jam si MA tangat circulum ACD, (*per 4. Corol. p. 1. h*) tangit sphæram: si verò neges MA esse tangentem circuli ACD, ducatur alia quævis IA quæ circulum ACD tangat in A: ergo [*per 32. l. 3.*] angulus IAD æquatur angulo C, id est [*per 5. l. 1.*] angulo D: ergo [*per 28. l. 1.*] IA & DC sunt parallelæ: sed MA, DC [*ex constr.*] sunt parallelæ: ergo [*per 30. l. 1.*] IA, MA sunt parallelæ, quod est absurdum, cum concurrunt in A. Ergo MA, & non IA, tangit circulum ADC.

LEMMA.

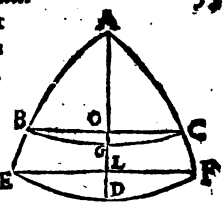
Si recta RG à centro ducta bisecet arcum BGC, bisecat ejus chordam BC, estque ad eam perpendicularis.

Demonstr. Quia arcus BG, GC sunt æquales, æquales sunt [*per 23. l. 1.*] anguli BRG, CRG. Æquantur etiam latera RB, RC, & latus RO est commune: ergo [*per 4. l. 1.*] bases BO, CO sunt æquales, adeoque bisecatur BC, quod erat primum] sicut & anguli ad O, qui proinde (*per 13. l. 1.*) sunt recti: ergo CO est perpendicularis ad RO, quod erat secundum.

PROPOSITIO XVI.

Si ex arcubus æqualibus [AE, AF] *auferantur arcus æquales* [AB, AC] *& per arcuum extremitates* [B & C, E & F] *ducantur circulorum maximorum arcus* [BGC, EDF] *Planum circuli maximi* [AD] *bisecantis angulum verticalem* [A] *bisecat arcus illi subtensos* [BGC, EDF] *& horum chordas* [BOC, EIF] *quæ etiam sunt parallelæ.*

Demonstr. I. Part. In triangulis BAG, CAG, anguli ad A



AG [*ex suppos.*] sunt æquales, sicut & latera AB, AC [*ex constr.*] latus verò AG est commune: ergo [*per 8. b*] bases BG, CG sunt æquales. Idem similiter ostendam de ED, FD: ergo bisecantur arcus BGC, EDF.

2. *Pars.* Linea à centro ducta & bisecans arcum BC in G, bisecat [*per Lemma præced.*] ejus chordam in O. Sed hæc linea est in plano circuli AD, quod [*ex suppos.*] transit per G, & (*per 1. Coral. 1. b*) per centrum sphæræ, id est [*per idem Coral.*] per centrum circuli BGC: ergo planum circuli AD bisecat chordam BOC. Eodem modo idem demonstratur de chorda ELF.

3. *Pars.* Sequitur ex dictis in prima parte angulos ad G esse æquales, adeoque (*per 2. b*) rectos: ergo planum circuli GC [*per 4. def. b*] rectum est plano circuli AGD. Deinde [*per Lemma præced.*] CO est perpendicularis lineæ ductæ à G ad centrum. Sed hæc linea est (*ut ostendimus in 4. b*) communis sectio circulorum AGD, BGC: ergo CO est perpendicularis sectioni communi planorum AG, CG; adeoque (*per 4. def. l. 11.*) est recta plano circuli AGD. Eodem ratiocinio ostendam FL esse rectam eidem plano AGD: ergo (*per 6. l. 11.*) lineæ CO, FL, adeoque & chordæ BOC, ELF sunt parallelæ.

CAPUT II.

De Resolutione Trianguli Sphærici Rectanguli.

PROPOSITIO I.

In omni triangulo sphærico rectangulo
Ut sinus cujusvis lateris, ad sinum anguli sibi oppositi:
Sic sinus alterius lateris, ad sinum anguli sibi oppositi.
Et è contrà.
Ut sinus cujusvis anguli, ad sinum lateris sibi oppositi:
Sic sinus alterius anguli, ad sinum lateris sibi oppositi.

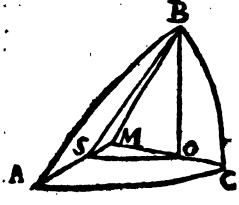
Hæc est prima regula pro resolutione trianguli sphærici rectanguli, & vocatur *Regula Sinuum seu Oppositionis.*

Nota Quando sermo est de triangulo rectangulo, nomine

Basis intelligi hypotenusam seu arcum oppositum angulo recto : arcus verò angulum rectum continentes simpliciter vocari *Latera*.

Proponatur triangulum sphæricum rectangulum ABC : dico sinum anguli recti C esse ad sinum basεos AB ; ut sinus cujusvis alterius ad basim anguli BAC, est ad sinum lateris oppositi BC.

Sit centrum sphæræ M, & sint MA, MB, MC communes sectiones circulorum AB, BC, AC. Ab angulo B ducantur BS, BO perpendiculares ad AM, CM ; & jungantur O & S rectâ OS. Ex hac constructione habetur



1.° Quòd BS (*per def. sinûs*) sit sinus arcûs BA, & BO sinus arcûs BC.

2.° Quia angulus C est rectus, planum MBC est (*per 4. def. c. 1. h*) rectum plano MCA : ergo (*per 4. def. l. 11.*) linea BO, quæ est sinus arcûs BC, adoque perpendicularis diametro MC, est recta plano MCA : ergo (*per 3. def. l. 11.*) angulus BOS est rectus.

3.° Quia BO est recta plano MCA, planum trianguli SBO ductum per BO est (*per 18. l. 11.*) rectum ad planum MCA : ergo cùm MS ponatur recta ad SB, est (*per 4. def. l. 11.*) recta ad planum trianguli SBO : ergo (*per 3. def. l. 11*) MS est etiam recta ad lineam SO : ergo (*per 6. def. l. 11.*) BSO est angulus quem faciunt plana circulorum BA, AC : ergo (*per 4. def. c. 1. h*) angulus BSO æqualis est angulo sphærico A : sicut & angulus rectus BOS angulo sphærico C. His præmissis

Demonstr. propositio. In triangulo plano BOS, recta BS (*per 1. c. 2. P. 1.*) est ad sinum totum, seu sinum anguli oppositi BOS, ut BO ad sinum anguli oppositi BSO. Sed BS, BO sunt sinus laterum BA, BC : & anguli BOS, BSO sunt æquales angulis sphæricis C & A : ergo BS sinus lateris BA, est ad sinum totum, seu sinum anguli oppositi C ; ut BO sinus alterius lateris BC, ad sinum oppositi anguli A : & è contrà.

Quod ostensum est de angulo acuto BCA facilè demonstratur de angulo obtuso BAÇ ; nam productis lateribus

BC, BA, fit novum triangulum ACD, cujus anguli & latera (*per. 2. Corol. 4. & 1. b*) ſunt priorum complementa; & eoſdem habent ſinus. Cùm igitur ſit in triangulo CAD, ut ſinus lateris AC ad ſinum anguli recti D; ita ſinus lateris CD ad ſinum anguli acuti CAD: erit etiam (propter ſinuum identitatem) in triangulo BAC, ut ſinus AC ad ſinum anguli recti B: ita ſinus lateris BC ad ſinum anguli obtuſi BAC.

Meliùs intelligetur tota hæc, uti & ſequens, demonſtratio, ſi ex charta, vel alia materia, circulorum ſegmenta confeceris, quæ ad angulos præſcriptos erecta, circulorum in ſphæra ſitum meliùs repræſentent.

PROPOSITIO II.

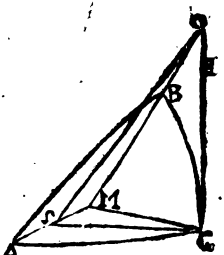
In omni triangulo ſphærico rectangulo
Ut ſinus totus, ad ſinum cujuſvis lateris circa angulum rectum;
Sic tangens anguli acuti prædicto lateri adjacentis,
Ad tangentem lateris ſibi oppoſiti.

Et è contrà.

Ut ſinus cujuſvis lateris circa angulum rectum;
Ad ſinum totum:
Sic tangens alterius lateris circa angulum rectum;
Ad tangentem anguli ſibi oppoſiti.

Hæc eſt ſecunda regula pro analyſi trianguli ſphærici, & vocatur *Regula Tangentium.*

In triangulo ABC ſit angulus C rectus. Sit M centrum ſphæræ, & MA, MB, MC ſectiones communes circulorum AB, AC, BC. Ab angulo C ducatur CS perpendicularis ad diametrum MA, & ſuper MC erigatur perpendicularis CO tangens circulum CB in C, & concurrens in O cum MB productâ. Demum jungantur puncta O & S.



Dico CS, ſinum lateris CA circa angulum rectum, eſſe ad ſinum totum, ut CO tangens alterius lateris A

circa angulum rectum, est ad tangentem anguli oppositi A. Et è contrà.

1.° Quòd SC sit sinus lateris CA, & CO tangens lateris CB, patet ex definitione sinûs & tangentis.

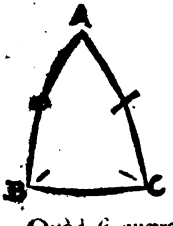
2.° Ostenditur (*ut in præced.*) triangulum planum SOC esse rectangulum in C: adeoque angulum SCO æqualem esse angulo sphærico C. Item OSC esse angulum quem faciunt plana circulorum BA, CA; adeoque æqualem angulo sphærico A. His præmissis

Demonstr. Propositio. In triangulo plano SOC, latus CS (*per p.* 2. *c.* 2. *P.* 1.) est ad sinum totum, ut CO ad tangentem anguli oppositi CSO. Sed CS est sinus lateris CA, & CO est tangens lateris CB, & anguli plani OCS, OSC sunt æquales sphæricis C & A: ergo CS sinus unius circa angulum rectum lateris CA, est ad sinum totum, ut CO tangens alterius lateris BC, ad tangentem anguli oppositi A; & è contrà.

PROPOSITIO III.

Resolvere triangulum sphæricum rectangulum.

1.° Si in triangulo proposito ABC detur oppositio, (*vg.* Si latere AB quæsito, cognoscantur angulus rectus B, basis AC, & angulus obliquus C oppositus lateri quæsito AB) adhiberi debet *Regula sinuum*, dicendo,



Ut sinus anguli noti B, *ad sinum lateris oppositi* AC:

Sic sinus anguli noti C, *ad sinum lateris quæsiti* AB.

Quòd si quæratur angulus, fiat,

Ut sinus lateris dati, ad sinum anguli oppositi:

Sic sinus alterius lateris dati, ad sinum anguli quæsiti.

Exemplum. Sit *vg.* anguli recti B sinus totus 10000000; basis AC gr. 70. 4'. ejusque sinus 9400899; angulus obliquus C 58 gr. ejusque sinus 8480481. Multiplicatis inter se secundo & tertio termino, productum (79724145352419) divide per primum terminum; & omissâ fractione, quo-

tientem (7972414) quære in tabulis sinuum; ibique invenies arcum seu latus quæsitum AB esse 52 gr. 52′.

Perficitur etiam operatio per sinus Logarithmicos eodem modo, quo in Trigonometria plana. Ratio dictorum patet *ex I. Prop. h.*

2.o Si non detur oppositio, cognoscantur verò duo latera AB, BC comprehendentia angulum rectum B; ex his notis, per *Regulam tangentium*, invenietur utervis angulus acutus *vg.* C: dicendo,

Ut sinus lateris dati BC, *ad sinum totum*:
Sic tangens alterius lateris dati AB, *ad tang. anguli oppositi* C.

Exemplum. Sit latus BC 41 gr. 18′. ejusque sinus 6600017; sinus totus 10000000; latus AB 71 gr. 55′. ejusque tangens 30625203. Quâ multiplicatâ per 10000000, productus 306252030000000 dividatur per 66000017; & erit quotiens 46401643 tangens anguli C gr. 77 50′.

Si verò detur unum latus CB juxta angulum rectum C, cum angulo acuto B adjacente lateri noto CB; hinc, per *Regulam tangentium*, exquiritur aliud latus AC, dicendo,

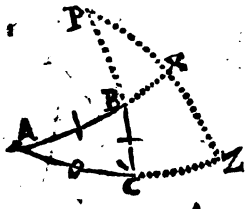
Ut sinus totus, ad sinum lat. dati CB:
Sic tangens anguli acuti B, *ad tang. lateris quæsiti* AC.

Exemplum per Logarithmos. Sinus totus Logarithmicus 100000000; latus CB sit 36 gr. 40′. ejusque sinus Logarithm. 97760897. Angulus B 53 gr. 20′. ejusque tangens Logarithm. 10128514. Secundo & tertio termino per additionem collectis in unam summam 199042411, ex ea subtrahe primum terminum 100000000; & erit residuum 99042411 quartus terminus, seu tangens Logarithmica lateris quæsiti AC; quam si exquiras inter Logarithmos tangentium, deprehendes arcum AC esse gr. 38. 44′. proximè.

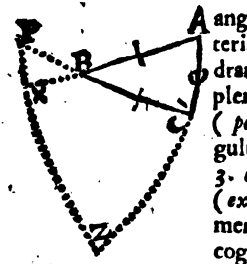
Utriusque partis ratio manifesta est ex 2 *prop. h.*

Nota, quòd quando per Regulam tangentium quæritur latus, primus terminus debet esse sinus totus; secundus verò sinus

lateris noti; tertius denique tangens anguli oppositi lateri quæsito. Si verò quæratur angulus, primus erit sinus lateris adjacentis angulo quæsito; secundus erit sinus totus; tertius tangens lateris oppositi angulo quæsito.



3.o Si ita disponantur termini dati, ut neutra regula locum habeat, sicut contingit in triangulo ABC, producantur (*ut in* 1. *vel* 2. *Fig.*) omnia trianguli dati latera, donec fiant quadrantes, & ducatur circuli maximi arcus PZ. In triangulo PBX (*per.* 7. *c.* 1. *h*) angulus X est rectus; latus BP est lateris noti BC complementum ad quadrantem; & latus BX est simile complementum lateris noti AB. Ex his (*per Regulam sinuum*) invenitur angulus P, adeoque arcus CZ, qui (*per.* 3. *c.* 1. *h*) est mensura anguli P, & (*ex constr.*) lateris quæsiti AC complementum ad quadrantem; quo noto, cognoscetur latus AC.



Ad eundem modum, quando denique nec per *Regulam sinuum* nec *tangentium* apparet solutio; si rectè producantur latera, dabitur in novo triangulo casus dato correspondens, semper per alterutram regulam resolubilis. Dixi, *si rectè producantur*, quia dubitari sæpe potest versùs quam partem fieri debeat hæc laterum productio; ad hoc autem determinandum juvabunt regulæ sequentes.

REGULÆ

Indicantes versùs quam partem cujusvis trianguli sphærici rectanguli arcus debeant produci.

1.a Si neuter acutus angulus notetur (id est, si neuter sit vel datus vel quæsitus) per quem libuerit angulum produci possunt arcus.

2.a Si unus tantùm angulus acutus notetur, produci debent arcus per angulum non notatum.

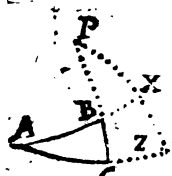
3.° Si uterque angulus acutus fuerit notatus, protrahantur arcus per angulum adjacentem alicui arcui notato.

Regulas has, & alia hactenus explicata meliùs intelliget Tyro ex Schemate subjecto, ubi exprimuntur casus omnes qui possunt in rectangulis contingere, & modus eorum quemvis unicâ operatione resolvendi.

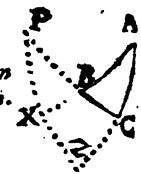
Nota 1.o literam Q in schemate significare *quæritur*. Et quamvis semper designetur aliquod latus, vel aliquis angulus quæsitus, eadem tamen solutio est alteri lateri vel angulo applicabilis.

Nota 2.o quòd si in triangulo rectangulo sint quævis duo latera majorá quadrante. (Hoc autem semper contingit, ut ostendam *cap.* 4, quando angulus aliquis est obtusus, vel latus aliquod quadrante majus) produci debeant latera majora donec fiant semicirculi, sic loco trianguli propositi habebitur novum triangulum resolvendum, cujus singula latera erunt quadrante minora, qualia requiruntur, ut per regulas datas perfici possit operatio.

Nota 3.o ut vitetur error, qui facilè potest contingere in accipiendis arcubus vel angulis pro eorum complementis ad semicirculum vel duos rectos; expedire ut, Tyrones præsertim, in charta vel tabula delineent triangula resolvenda, eorumque latera producant ex præscripto regularum; hinc enim fiet ut totius resolutionis schemate oculis subjecto, faciliùs possint observare, quomodò ex inventis innotescant quæsita.



SCHEMA I.



Pro analysi triangulorum sphæricorum rectangulorum.

Quæritur latus aliquod *a c* vel *b c*.

1	*Ex ang oppos.* b, & *basi* ab, Q. a c.	*Ut sinus totus, ad sinum baseos* a b. *Sic sinus ang. dati* b. *ad sinum lateris* a c.
2	*Ex ang. adjac.* a, & *basi* a b, Q. a c.	*Ut sin. compl. ang. dati* a, *ad sin. totum*: *Sic tang. compl. baseos* ab, *ad tang. compl. lat.* a c.
3	*Ex altero lat* b c. & *basi* a b, Q. a c	*Ut sin. compl lat dati* b c, *ad sin. totum*: *Sic sin compl bas* a b *ad sin. compl. lat.* a c.
4	*Ex ang. oppos.* b. & *altero lat.* b c, Q. a c.	*Ut sin. tot. ad sinum lat dati* b c: *Sic tang. ang. dati* b. *ad tang lat.* a c.
5	*Ex ang adjac.* a, & *altero lat.* b c, Q. a c.	*Ut tang. ang. dati* a. *ad tang lat. dati* b c: *Sic sin. tot. ad sinum lat.* a c.
6	*Ex omnibus angulis.* Q. b c.	*Ut sin. ang. adjac.* b, *ad sin. compl. ang. oppos.* a: *Sic sin. tot. ad sin. compl. lat.* b c.

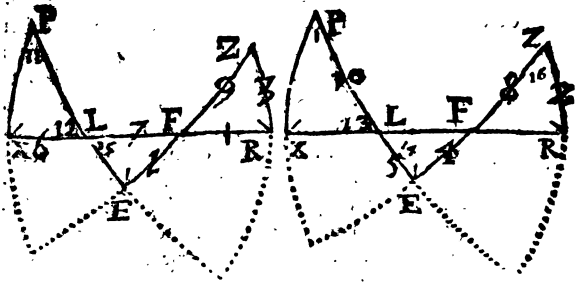
Quæritur Basis *a b*.

7	*Ex quovis latere* b c, & *ang. ei oppos.* a.	*Ut sin. ang. dati* a, *ad sin lat. dati* b c: *Sic sin. tot. ad sin. baseos* a b.
8	*Ex quovis lat.* a c, & *ang. ei adjac.* a.	*Ut sin. tot. ad sin. compl. ang. dati* a: *Sic tang compl lat. dati* a c, *ad tang compl. bas.* a b.
9	*Ex utroque latere* a c & b c.	*Ut sin. tot. ad sin. compl. lat.* b c: *Sic sin. compl. lat.* a c. *ad. sin. compl bas.* a b.
10	*Ex omnibus angulis.*	*Ut tang. acuti* b, *ad tang. compl. acuti* a: *Sic sin. tot. ad sin. compl. bas.* a b.

Quæritur angulus aliquis *a* vel *b*.

11	*Ex latere oppos.* b c, & *altero acuto* b Q. a.	*Ut sin. tot. ad sin. compl. lat. dati* b c: *Sic sin. ang. dati* b, *ad sin. compl. ang.* a,
12	*Ex latere adjac.* b c, & *altero acuto* a, Q. b.	*Ut sin. compl. lat. dati* b c *ad sin tot.* *Sic sin. compl. ang. dati* a, *ad sin. ang. quæsiti* b.
13	*Ex basi* a b & *altero acuto* a, Q. b.	*Ut sin compl. bas* a b, *ad sin tot*: *Sic tang. compl. ang. dati* a, *ad tang. ang.* b.
14	*Ex utroque latere* a c, b c.	*Ut sin lat. adjac. quæsito, ad sinum tot*: *Sic tang. ulterius lat. ad tang. ang. quæsiti.*
15	*Ex latere oppos.* & *basi.*	*Ut sin baseos, ad sin. tot*: *Sic sin. lat. oppos. ad sin. ang. quæsiti.*
16	*Ex lat. adjac.* a c, & *basi* a b, Q. a.	*Ut tang. compl lat. dati* a c, *ad tang. compl. bas.* a b: *Sic sin. tot. ad sin. compl. ang.* a.

Ad



Ad laterum productionem, & quicquid schemate hoc explicatur, meliùs percipiendum plurimùm proderunt figuræ appositæ, in quibus arcus omnes triangulorum LEF in utramque partem producuntur donec utrinque fiant quadrantes, adeoque & anguli E, X, R (*per.* 7. *c.* 1. *b*) recti. Notæ verò, & numeri casum quemlibet exprimentes, ita disponuntur, ut

1.o *primæ Figuræ* contineat omnes casus sinuum ope resolubiles; *secunda* illos, ad quorum solutionem tangentes adhibentur. Unde primo intuitu scias, utrùm *per Regulam sinuum*, an verò *tangentium*, ullus casus oblatus debeat resolvi.

2.o Notæ & numeri lateribus vel angulis appositi ostendunt, quid in unoquoque casu, & ex quibus datis quæratur; nam numerus casum exprimens semper adscribitur lateri vel angulo per illum casum quæsito: reliquæ verò notæ designant latera & angulos, ex quibus datis quæritur solutio. *vg.* in triangulo FZR (*in Fig.* 1.) numerus 9 adscriptus arcui seu basi FZ, indicat quæri in nono casu basim FZ: reliquæ verò notæ appositæ lateribus FR, ZR, & angulo recto R signant data, ex quibus in illo casu basis FZ quæritur. Similiter numerus 3 adhærens lateri ZR ostendit quòd in tertio casu quæratur latus aliquod ZR ex reliquis notatis; viz: ex basi FZ, latere FR, & angulo recto R. Quia verò uterque casus habetur *in* 1. *Fig.* inde colliges utrumque solvi *per Regulam sinuum*.

3.o Casus omnes, qui resolvuntur sine laterum productione, exprimuntur in triangulis intermediis LEF. Sic quia angulo ELF inscribitur (*in Fig.* 2) numerus 14. hinc infertur quòd in casu decimo-quarto, *per Regulam tangentium*, & absque laterum productione, quæratur aliquis angulus acutus ELF, ex lateribus notatis EL, EF, & angulo recto E notato.

4.o In cornibus XPL, FZR casus omnes arcuum protractione indigentes ita disponuntur, ut in triangulis intermediis semper habeatur propositi casûs solutio. *Ex. gr.* in triangulo XPL (*in Fig.* 1.) quæratur, ut in sexto casu, ex omnibus angulis notatis latus XL. 1.o Quia notatur uterque angulus acutus, productio fiet (*juxta Regulam* 3.) per angulum L adjacentem lateri notato XL. 2.o In triangulo LEF cognoscuntur angulus ELF æqualis dato XLP, & latus EF complementum anguli dati P. & angulus rectus E; ex quibus (*per Reg. sinuum*) invenitur basis LF, complementum lateris quæsiti XL.

CAPUT III.

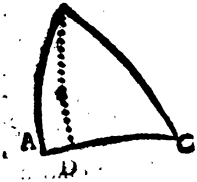
De analysi trianguli sphærici obliquanguli.

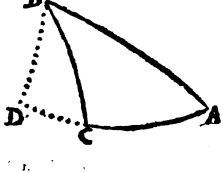
PROPOSITIO I.

In omni triangulo sphærico obliquangulo est
Ut sinus cujusvis lateris, ad sinum anguli sibi oppositi:
Sic sinus cujusvis alterius lateris, ad sinum anguli sibi oppositi.

Et è contrà.

Ut sinus cujusvis anguli, ad sinum lateris sibi oppositi:
Sic sinus cujusvis alterius anguli, ad sinum lateris sibi oppositi.





Hæc vocatur *Regula Sinuum* seu *Oppositionis*, & locum habet in triangulis obliquangulis, quandocunque invenitur Oppositio.

Demonstr. Ab aliquo angulo B ducatur arcus BD per

pendicularis ad latus oppositum AC, si opus fuerit productum. Sive BD cadat intra, sive extra triangulum ABC, erit in triangulo rectangulo ABD (*per.* 1. *c.* 1. *h*) sinus lateris AB ad sinum totum, ut sinus lateris BD ad sinum anguli A: & in triangulo rectangulo BCD, erit (*per eandem*) sinus totus ad sinum lateris BC, ut sinus anguli BCD ad sinum lateris BD. Ergo

1. *Sinus lat.* AB. 3. *Sinus ang.* BCD.
2. *Sinus totus* 1. *Sinus lat.* BD.
3. *Sinus lat.* BC. 2. *Sinus anguli* A.

(*ex æquo perturbato*) ut sinus lateris AB, ad sinum lateris BC; ita sinus anguli BCD, ad sinum anguli A. Et (*permutando*) ut sinus lateris AB ad sinum anguli BCD seu BCA (idem enim est utriusque sinus:) ita sinus lateris BC ad sinum anguli A; & (*invertendo*) ut sinus anguli BCA ad sinum lateris AB: ita sinus anguli A, ad sinum lateris CB.

PROPOSITIO II.

Resolvere triangulum sphericum obliquangulum, in quo datur oppositio.

1.° Si quæratur angulus aliquis A, fiat

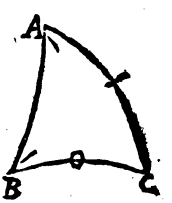
Ut sinus lateris dati AC, *ad sinum anguli oppositi* B:

Sic sinus lateris noti BC, *ad sinum anguli quæsiti* A.

2.° Si quæratur latus aliquod BC, fiat

Ut sinus anguli dati B, *ad sinum lateris oppositi* AC.

Sic sinus anguli dati A, *ad sinum lateris quæsiti* BC.



Ratio utriusque partis patet *ex præcedente.*

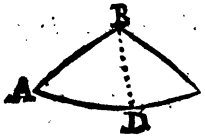
PROPOSITIO III.

Resolvere triangulum obliquangulum in qua nulla reperitur oppositio.

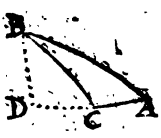
Quando nulla est oppositio, demitti debet ab aliquo angulo arcus perpendicularis, qui triangulum obliquangulum datum resolvat in duo triangula rectangula, quæ per regulas suprà traditas resoluta dabunt latera & angulos quæsitos.

In demittendo autem arcu perpendiculari duo sunt observanda; nam 1.o duci debet ab uno extremo alicujus lateris cogniti, cujus alteri extremo adjacet angulus etiam notus. 2.o Cavendum est ne demittatur ab angulo signato ad latus signatum. Defectu hujus cautionis, solutio fiet satis difficilior; defectu verò prioris, evadet impossibilis.

Hæc resolutio fieri potest per tres, vel etiam per duas tantùm operationes. Primus modus licèt operosior, facilius à Tyronibus intelligetur; in quorum gratiam illius schema subjicio, exhibens omnium casuum oppositione carentium (duobus demptis, de quibus *Prop. 4. & 5. h*) solutionem, & docens ex quo angulo duci debeat perpendicularis, & quo ordine reliqua sint peragenda. Observari tamen debet quòd in schemate subjecto linea BD semper significet arcum perpendicularem; & literæ *c*, casus, qui citantur ex I. schemate.



SCHEMA SECUNDUM.



Pro resolutione triangulorum sphæricorum obliquangulorum.

Latus quæritur.

1 *Ex 2 lat.* a b, a c, *& ang.* [*Primò. Quære* (ut in. 1. c.) b d. *Secundò* (ut a *ab eis comprehenso*, [in. 2. vel 3. c.) a d, *notumque erit* d c. *Un-* l *de tertiò* (ut in. 9. c.) *invenitur* b c. b c.

2 *Ex 2 lat.* b a, b c, *&* [*Primò. Quære* (ut. in 1. c.) bd. *Secundò* (ut *ang.* c *adjacente lat. quæ-* [in. 3. c.) a d. *Tertiò* (ut in 3. c.) d c, *n-* *sito* a c. l *tumque erit* a c *quæsitum.*

3 *Ex lat.* b c, *& 2 ang.* a [*Primò. Quære* (ut in. 1. c.) b d. *Secundò* (ut *&* c *adjacentibus lateri-* [in. 5. c.) a d. *Tertiò* (ut in. 2. vel 3. c.) c d; *quæsito* a c. l *notumque erit* a c *quæsitum.*

4 *Ex 2 ang.* b *&* c, *&* [*Primò. Quære* (ut in. 1. c.) b d. *Secundò* (ut *latere contento* b c, [in. 12. 13. vel 16. c.) *ang.* c b d. *notusque* a b. l *erit* a b d. *Tertiò* (ut in. 8. c.) a b *quæsit.*

Angulus quæritur.

5 *Ex 2 ang.* b *&* c, *&* [*Primò. Quære* (ut in. 1. c.) b d. *Secundò* (ut *lat. contenta* b c, [in. 13. c.) *ang.* d b c. *notusque erit* d b a. *Ter-* a. l *tiò* (ut in. 11. c.) *ang.* b *quæsit.*

6 *Ex 2 ang.* a & c, *& lat.* a b *adjacente ang. quæsit.* b. | *Primò. Quare* (ut in. 1. c.) b d. *Secundò* (ut in. 13. c.) *ang.* a b d. *Tertiò* (ut in. 12. c.) *ang.* c b d. *notusque erit* a b *& quæsit.*

7 Ex 2 *lat.* a b, a c, *& ang.* a *comprehenso*, *&* b c a. | *Primò. Quare* (ut in. 1. c.) b d, *Secundò* (ut in. 2. c.) a d, *notumque erit* c d. *Tertiò* (ut in. 14. c.) *ang* b c d *seu* b c a.

8 *Ex ang.* c, *& 2 lat.* a b, b c *continentibus ang.* b *quæsitum* b. | *Primò. Quare* (ut in. 1. c.) b d. *Secundò* (ut in. 16. c.) *ang.* a b d. *Tertiò* (ut in. 13. vel 16. c.) *ang* c b d, *notusque erit* a b c *quæsito.*

In sequenti schemate indicatur modus resolvendi prædictæ triangula per duas operationes, & sua cuique casui demonstratio subjungitur.

Nota BD ubique esse perpendiculum; & quando casus citantur, recurrendum esse ad schema I, ubi invenientur casus citati, sed interdum ordine terminorum permutato.

SCHEMA III.

Pro resolutione triangulorum sphæricorum obliquangulorum.

Latus quæritur.

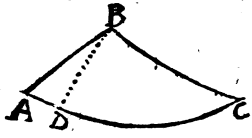
1 *Ex 2 lat.* ab, ac, *& ang.* 2 *ab eis compræhenso*, 2 b c. | 1.o *Quare* (ut in. 2. c.) a d. 2. b c, *quod fit dicendo*, 1 *Ut sin. compl. lat.* a d, *ad sin. compl. lat.* a b: 1 *Sic sin compl. lat.* c d, *ad sin. compl. lat. quæsiti* b c.

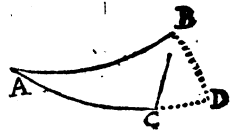
Cætera per se patent; ostendi solùm debet quòd legitima sit proportio assumpta.

Demonstr. Ut sinus compl. lat. *a d*, ad sinum compl. lat. *a b*:
Sic (*per.* 3. *c*) sinus totus, ad sinum compl. lat. *b d*.
Et, Ut sinus compl. lat. *c d*, ad sinum compl. lat. *b c*:
Sic (*per.* 3. *c.*) sinus totus, ad sinum compl. lat. *b d*.
Ergo (*per.* 11. *l.* 5) Ut sinus compl. lat. *a d*, ad sinum compl. lat. *a b*:
Sic sinus compl. lat. *c d*, ad sinum compl. lat. *b c*.

2 *Ex 2 lat.* a b, b c, *& ang.* 2 *adjac. lat. quæsito* a c. | 1.o *Quare* (ut in. 2. c.) a d. 2. d c, *dicendo*, 1 *Ut sin. compl. lat.* a b, *ad sin. compl. lat.* a d: 1 *Sic sin. compl. lat.* b c, *ad sin. compl. lat.* d c.

Demonstr. Ut sinus compl. lat. *b d*, ad sinum totum:
Sic (*per.* 3. *c.*) sinus compl. lat. *a b*, ad sinum compl. lat. *a d*.
Et, Ut sinus compl. lat. *b d*, ad sinum totum:
Sic (*per.* 3. *c.*) sinus compl. lat. *b c*, ad sinum compl. lat. *d c*.
Ergo (*per.* 11. *l.* 5) Ut sinus compl. lat. *a b*, ad sinum compl. lat. *a d*:
Sic sinus compl. lat. *b c*, ad sinum compl. lat. *d c*.





3 *Ex lat.* a b, & 2 l 1.o *Quære* (ut in. 2. c.) a d. 2. d c, *dicendo*,
ang. a & c *adjacen.* | *Ut tang. ang* c, *ad tang. ang.* a:
lat. quæſit. a c. | *Sic ſin. lat.* a d, *ad ſin. lat.* d c.

Demonſtr. Ut tang. ang. *c*, ad tang. lat. *b d*:
Sic (*per.* 5. *c.*) ſinus totus, ad ſinum lat. *d c*.
Et, Ut tang. lat. *b d*, ad tang. ang. *a*:
Sic (*per.* 14 *c.*) ſinus lat. *a d*, ad ſinum totum.
Ergo (*per.* 23. *l.* 5) Ut tang. ang. *c*, ad tang ang. *a*:
Sic ſinus lat. *a d*, ad ſinum lat. *d c*.

4 *Ex lat.* a b, & 2 l 1.o *Quære* (ut in. 13. c.) *ang.* a b d. 2. b c, *dicendo*,
ang a & b *adjac.* | *Ut ſin. compl. ang.* a b d, *ad tang. compl. lat.* a b:
dato lat. a b, & b c. | *Sic ſin. compl ang.* c b d, *ad tang. compl. lat.* b c.

Demonſtr. Ut ſinus totus, ad tang. compl. lat. *b d*:
Sic (*per.* 8. *c.*) ſinus compl. ang. *a b d*, ad tang. compl. lat. *a b*.
Et, Ut ſinus totus, ad tang. compl. lat. *b d*:
Sic (*per.* 8. *c.*) ſinus compl. ang. *c b d*, ad tang. compl. lat. *b c*.
Ergo (*per.* 11. *l.* 5) Ut ſinus compl. ang. *a b d*, ad tang. compl. lat. *a b*:
Sic ſinus compl. ang. *c b d*, ad tang. compl. lat. *b c*.

Angulus quæritur.

5 *Ex* 2 *ang.* b & a, & l 1.o *Quære* (ut in. 13. c.) *ang* a b d. 2. b c a, *dicendo*,
lat. a b *ab eis in-* | *Ut ſin. ang.* a b d, *ad ſin. compl. ang.* a:
tercepto 2 c. | *Sic ſin. ang.* c b d, *ad ſin. compl. ang.* c.

Demonſtr. Ut ſinus totus, ad ſinum compl. lat. *b d*:
Sic (*per.* 11. *c.*) ſinus ang. *a b d*, ad ſinum compl. ang. *a*.
Et, Ut ſinus totus, ad ſinum compl. lat. *b d*.
Sic (*per* 11. *c.*) ſinus ang *c b d*, ad ſinum compl. ang. *c*.
Ergo (*per.* 11. *l.* 5) Ut ſinus ang. *a b d*, ad ſinum compl. ang. *a*:
Sic ſinus ang. *c b d*, ad ſinum compl. ang. *c*.

6 *Ex 2 ang.* a & c, & 1 l.o *Quare* (ut in. 13. c.) a b d. 2. a b c, *dicendo*, *lat.* a b *adjac. ang.* 1 *Ut sin compl. ang* a, *ad sin compl. ang.* c: *quæsit.* b. 1 *Sic sin. ang.* a b d, *ad sin. ang.* c b d.

Demonstr. Ut sinus compl. lat. *b d*, ad sinum totum:
Sic (*per* 12. *c.*) sinus compl. ang. *a*, ad sinum ang. *a b d.*
Et, Ut sinus compl. lat. *b d*, ad sinum totum:
Sic (*per.* 12. *c.*) sinus compl. ang. *c*, ad sinum ang. *c b d.*
Ergo (*per.* 11. *l.* 5) Ut sinus compl. ang. *a*, ad sinum ang. *a b d.*
Sic sinus compl. ang. *c*, ad sinum ang. *c b d.*

7 *Ex 2 lat.* a b, a c, & 1 l.o *Quare* (ut in. 2. c.) a d. 2. *ang.* c, *dicendo*, *& ang.* a *ab eis contento*, Q. c. 1 *Ut sin. lat.* c d, *ad sin. lat.* a d: 1 *Sic tang. ang.* a, *ad tang. ang.* c.

Demonstr. Ut sinus lat. *c d*, ad sinum totum:
Sic (*per.* 14. *c.*) tang. lat. *b d*, ad tang. ang. *c.*
Et, Ut sinus totus, ad sinum lat. *a d:*
Sic (*per.* 4. *c.*) tang. ang. *a*, ad tang. lat. *b d.*
Ergo (*per.* 23. *l.* 5) Ut sinus lat. *c d*, ad sinum lat. *a d:*
Sic tang. ang. *a*, ad tang. ang. *c.*

8 *Ex ang.* a, *& 2 lat.* 1 l.o *Quare* (ut in. 13. c.) *ang.* a b d. 2. c b d, *dicendo*, a b, b c *comprehen-dentib. ang. quæsit.* b 1 *Ut tang compl. lat.* a b, *ad sin. compl. ang.* a b d: 1 *Sic tang. compl lat.* b c, *ad sin. compl. ang.* c b d.

Demonst. Ut tang. compl. lat. *b d*, ad sinum totum:
Sic (*per.* 16. *c.*) tang. compl. lat. *a b*, ad sinum compl. ang. *a b d.*
Et, Ut tang. compl. lat. *b d*, ad sinum totum:
Sic (*per.* 16. *c.*) tang. compl. lat. *b e*, ad sinum compl. ang. *c b d.*
Ergo (*per.* 11. *l.* 5) Ut tang. compl. lat. *a b*, ad sinum compl. ang. *a b d.*
Sic tang. compl. lat. *b c*, ad sinum compl. ang. *c b d.*

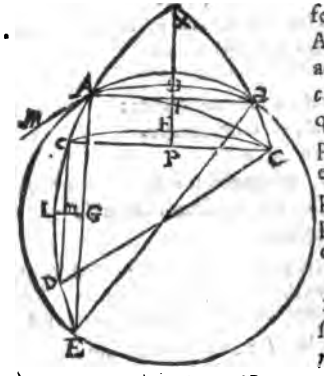
LEMMA AD PROP. IV.

In omni triangulo spherico, rectangulum sub sinubus crurum, est ad quadratum radii: ut rectangulum sub sinubus differentiarum crurum à semisumma omnium laterum, Ad quadratum sinûs dimidii anguli verticalis.

Proponatur triangulum BCA; sit angulus quæsitus & verticalis B; crus minus BA, crus majus BC; basis AC.

CONSTRUCTIO.

Ex crure majori BC auferatur B *a* arcus æqualis cruri minori BA, & producatur BA ad *c*, ut adæquet BC. Puncta A *a* & *c* C jungantur arcubus circulorum maximorum AO *a* & *c* FC, quibus subtendantur chordæ AI *a*, *c* PC: angulum verticalem B bisecet planum circuli maximi BOF, bisecabit (*per.* 16. *c.* 1. *b*) etiam arcus AO *a*, *c* FC & eorum chordas.



Dein trianguli basis AC transferatur in circulum BA *c* ex A usque ad D, & ex *c* usque ad E, subtensis chordis AE, *c* D, quæ simul cum arcubus, quos subtendunt, bisecentur per rectam LNG. Ducantur etiam rectæ E *a*, DC & huic parallela MA. Denique per puncta EA *a* describatur circulus. In hac constructione

1.o Chordæ A *a*, *c* C (*per* 16. *c.* 1. *b*) sunt parallelæ, sicut & AE, *c* D (*per* 1. *Corol. p.* 29. *l.* 3.)

2.o Arcus BL est semisumma omnium laterum; nam crus minus AB duplicatur per superimpositionem majoris BC; & residuum cruris majoris BC, viz: A *c*, duplicatur per superimpositionem baseos AC ex A in D; & residuum baseos AC, viz: *c* D, duplicatur in ejus dimidio *c* L.

3.o Arcus AL est differentia inter crus minus BA & BL semisummam omnium laterum. Et AG est sinus illius differentiæ; nam (*per Lemma ante* 16. *c.* 1 *b*) est perpendicularis lineæ LNG quæ à centro ducta bisecat arcum ALE, ejusque chordam AGE.

Similiter arcus *c* L est differentia inter crus majus BC (seu B *c*) & BL semisummam omnium laterum, & *c* N est sinus illius differentiæ.

4.o Recta I *a* est sinus arcûs O *a*, & PC sinus arcûs BC; nam

nam (*per Lemma ante* 16. *c.* 1. *h*) sunt perpendiculares dimetris bisecantibus arcus AO *a*, *c*FC in O & F.

5to Angulus X est dimidium anguli verticalis ABC; quippe quem (*ex constr.*) bisecat arcus BOP.

Ostendi igitur debet rectangulum sub sinubus crurum BA, BC, esse ad quadratum radii, ut rectangulum sub AG & *c*N, est ad quadratum sinûs anguli X.

DEMONSTRATIO.

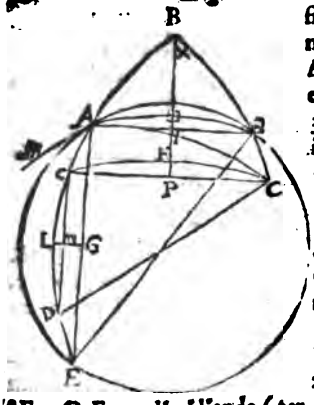
In triangulo sphærico B*a* O, quod (*ut patet ex* 8 & 16. *c.* 1. *h*) est rectangulum in O, sinus baseos B*a* est (*per* 1. *c.* 2. *h*) ad sinum totum: ut I*a* sinus lateris O*a*, ad sinum anguli oppositi X.

Similiter in triangulo sphærico rectangulo BCF sinus baseos BC est ad sinum totum, ut PC sinus lateris FC ad sinum anguli oppositi X.

Multiplica nunc inter se utriusque analogiæ terminos, & erit (per *Corol. p.* 22. *l.* 5) rectangulum sub sinubus crurum B*a* (seu BA) & BC, ad quadratum sinûs totius, ut rectangulum sub I*a*, PC, ad quadratum sinûs anguli X.

Quòd primus, secundus, & quartus terminus hujus analogiæ iidem sint, qui in Lemmate propositi, patet *ex constr.* Quòd autem tertius æquetur tertio, id est, rectangulum sub I*a*, PC, rectangulo sub AG, *c*N, sic ostendo.

Quia parallelæ sunt A*a* & *c*C, item AE & *c*D, æquales sunt (*per* 10. *l.* 11.) anguli *a*AE & C*c*D, & (*per* 15. *l.* 11.) plana triangulorum AE*a*, CD*c* per istas lineas ducta sunt parallela. Deinde quia triangulum sphæricum ADC est (*ex constr.*) Isoscele, recta MA (*ex constr.*) parallela chordæ DC (*per* 15. *c.* 1. *h*) tangit sphæram in puncto A; & quia tangit planum trianguli AE*a* in puncto A, & parallela est chordæ DC existenti in plano *c*CD parallelo priori AE*a*, tota MA (*per Corol. p.* 16. *l.* 11.) est in eodem plano cum triangulo AE*a*: ergo, quia triangulum AE*a* est in eodem plano cum circulo AE*a*, MA est etiam in eodem plano: ergo MA (*per* 15. *c.* 1.) tangens sphæram in A, tangit etiam circulum AE*a* in eodem puncto A; cùm enim tota circumferentia circuli AE*a* sit (*per* 2. *Corol. p.* 1. *c.* 1. *h*) in superficie



ficie ſphæræ, patet quòd MA non poſſit ſecare circulum AE *a*, quin ſecet ſphæram: ergo angulus MAE eſt (*per* 32. *l.* 3) æqualis angulo A *a* E in ſegmento alterno. Sed quia MA, DC, & AE, *c*D ſunt parallelæ, anguli MAE, CD*c* (*per* 10. *l.* 11) ſunt æquales: ergo anguli A *a* E, CD*c* ſunt æquales; adeoque in triangulis AE *a*, C *c* D æquales ſunt anguli *a* AE, C *c* D, item A *a* E, CD*c*: ergo (*per* 2. *Corol. p* 4. *l.* 6) A*a*: *c*D:: AE: *c*C. Ergo dimidiando (*per* 15. *l.* 5) I*a*: *c*N:: AG: PC: ergo (*per* 16. *l.* 6) rectangulum ſub extremis I*a*, PC, æquatur rectangulo ſub mediis AG, *c*N.

PROPOSITIO IV.

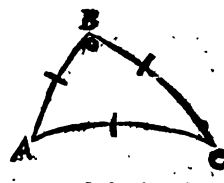
Datis tribus lateribus invenire angulos.

1.o Si latus aliquod AB ſit quadrans, aliud latus AC vel AE producatur vel minuatur ad quadrantem AD, & ducatur arcus BD. In triangulo CBD, vel EBD ſcitur latus CD vel DE differentia dati AC vel AE à quadrante AD, & latus BC vel BE datum: angulus verò D (ſicut & ABD) eſt rectus, (ut oſtendam *prop.* 6. *c.* 4. *h.*) Ex his invenietur arcus BD, qui (*per* 3. *c.* 1. *h*) eſt menſura anguli A.

2.o Si proponatur triangulum Iſoſcele ABC, ab angulo A inter æqualia latera contento, ad baſim BC biſectam in D, ducatur arcus AD. Jam quia triangula ABD, ACD ſunt (*ut patet*) ſibi mutuò æquilatera, anguli ad D ſunt (*per* 14. *c.* 1. *h*) æquales; adeoque (*per* 2. *c.* 1. *h*) recti. Ergo in triangulo ABD cognoſcuntur la-

tera AB, BD cum angulo recto D. Ex his (*per* 3. *c.* 2. *b*) noti fient anguli B & BAD, quibus (*per* 14. *c.* 1. *b*) æquantur anguli C & CAD.

3.° Extra casus prædictos, fiat (si quæratur *v. g.* angulus B) ut rectangulum sub sinubus crurum AB, CB, ad quadratum radii: sic rectangulum sub sinubus differentiarum crurum AB, CB à semisumma omnium laterum, ad aliud; & erit (*per Lemma*) quartus terminus quadratum sinûs dimidii anguli quæsiti B.



Nota angulum quæsitum semper accipiendum esse pro angulo verticali, & latus oppositum pro basi.

Ut melius intelligatur casûs hujus solutio, praxim tum per sinus naturales, tum per Logarithmos exhibebo. Sit igitur in triangulo proposito ABC crus minus BA 39. gr. 20'. crus majus BC 60 gr. basis AC 78 gr. 30'.

Ordo resolutionis per sinus naturales.

1.° Multiplicando inter se sinus naturales crurum BA, BC invenitur rectangulum sub sinubus crurum, quod est primus terminus.

2.° Ducendo sinum totum in seipsum, habetur quadratum radii, quod est secundus terminus.

3.° Subtracto crure BA ex semisumma omnium laterum; quære sinum residui seu differentiæ: similiter subtracto crure BC ex eadem semisumma, quære sinum differentiæ. Hos differentiarum sinus inter se multiplica, & prodibit rectangulum sub sinubus differentiarum crurum à semisumma omnium laterum, quod est tertius terminus.

4.° Ducatur secundus terminus in tertium, & productus dividatur per primum; erit quotiens quadratum sinûs dimidii anguli B; hujus radicem extrahe, notaque erit dimidius, adeoque totus angulus B.

Forma calculi per Logarithmos.

2.° Sinus Logarithmicos { lat: BA --- 98019735. & lat: BC -- 99375306. } Adde.

Erit ſumma --- 197395041.

Rectangulum ſub ſinubus crurum, & primus terminus.

2.° Duplica Logarithmum ſinûs totius --- 100000000.

ut habeas quadratum radii ------ 200000000.

quod eſt ſecundus terminus.

3.° Latera { BA. 39 gr. -- 20′ / BC 60 --- 00 / AC 78 --- 30 } Adde ut habeatur

Laterum ſumma. 177 --- 50 &

Semiſumma ---- 88 --- 55 hinc ſubtrahe

Latus BA --- 39 --- 20

Reſiduum --- 49 --- 35 eſt prima differentia.

Ex eadem ſemis. 88 --- 55 ſubtrahe

Latus BC --- 60 --- 00

Reſiduum --- 28 --- 55 eſt ſecunda differentia.

Differentiarum Logarithmicos ſinus { 98815842 / 96844297 } Adde

Erit ſumma --- 195660139 Rectangulum ſub ſinubus differentiarum, quod eſt tertius terminus.

4.° 2 Terminum --- 200000000 } Adde

& 3 Terminum 195660139 }

Ex ſumma --- 395660139 } Subtrahe

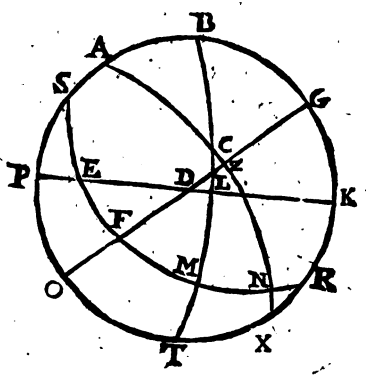
1 Terminum 197395041 }

Reſiduum ---- 198265098 eſt quadratum ſinûs dimidii anguli B; ac proinde radix ſeu ſemiſſis ejus 99132549 eſt ipſe ſinus, qui in tabulis inventus dat 54 gr. 59′, pro dimidio anguli B; adeoque totus angulus B continet 109 gr. 58′.

LEMMA AD PROP. V.

Dato quocunque triangulo ſpherico (ABC) *eſt in ſphera aliud triangulum* (DEF) *cujus ſingula latera* (DF, DE, EF) *æquantur ſingulis angulis* (A, B, C:) *& ſinguli anguli* (D, E, F,) *ſingulis lateribus* (AB, BC, AC:) *mutato tamen utrinque angulo maximo v.g.* B *& F in ſuum complementum ad duos rectos.*

Perficiatur circulus AB, cui occurrant producti AC in X, & BC in T. Deinde punctis A, B, C, tanquam polis seu centris, intervallo quadrantis describantur circuli GO, KP, RS. Hinc



1.o Quia A est polus circuli GO, anguli G, Z (*per 6. c. 1. h*) sunt recti: similiter quia B est polus circuli KP, recti sunt anguli K, L; & quia C est polus circuli RS, recti sunt anguli M, N.

2.o Quia anguli G & K sunt recti, arcus GD, KD transeunt (*per 6. c. 1. h*) per polum circuli GK, qui proinde est D, adeoque (*per 5. c. 1.*) GD, KD sunt quadrantes. Similiter quia recti sunt anguli Z, N, arcus ZF, NF concurrunt in polo F, & sunt quadrantes. Denique quia anguli L & M sunt recti, arcus ME, LE concurrunt in polo E, & sunt quadrantes.

Demonstr. jam I. Pars. Et 1.o Quia quadrantes sunt LE, KD, sublato communi DL, restant DE, KL æquales. Sed (*per 3. c. 1. h*) KL, adeóque & DE, est mensura anguli KBL, qui complet angulum maximum B ad duos rectos. Ergo.

2.o Quia quadrantes sunt ZF, GD, sublato communi ZD, restat DF æqualis arcui GZ, qui est (*per 3. c. 1. h*) mensura anguli BAC.

3.o Quia quadrantes sunt ME, NF, sublato communi MF, restat EF æqualis arcui MN, qui est mensura anguli MCN, id est (*per 2. c. 1. h*) anguli ACB.

2. *Pars.* 1.o Quia quadrantes sunt CN, AZ, sublato communi CZ, restat AC æqualis arcui ZN, mensuræ anguli ZFN, qui complet angulum maximum EFD ad duos rectos.

2.o Quia quadrantes sunt CM & BL, ablato communi CL, restat LM mensura anguli FED, & æqualis arcui BC.

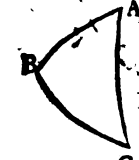
3.o Quia AG & BK sunt quadrantes, subtracto communi BG, restat AB æqualis arcui GK, qui est mensura anguli GDK, id est (*per 2. c. 1. h*) anguli EDF.

PROPOSITIO V.

Datis tribus angulis invenire latera.

Proponatur triangulum ABC, in quo angulus B sit v. g. 109 gr. 58′: A 56 gr. 10′. C 37 gr. 26′.

Anguli maximi B complementum ad duos rectos, viz: 70 gr. 2′, adscribe lateri opposito AC: lateri BC appone 56 gr. 10′ quantitatem anguli oppositi A: & lateri AB 37 gr. 26′ quantitatem anguli oppositi C. Angulis sic in latera opposita translatis, (*per præced.*) ex tribus lateribus datis invenies angulum B esse 101 gr. 30′. Accipe ejus complementum ad duos rectos, viz: 78 gr. 30′. erit hæc vera quantitas lateris oppositi AC.



Nota quòd invento latere ex tribus angulis, vel uno angulo ex tribus lateribus, reliqua facilè investigentur per Regulam sinuum.

CAPUT IV.

De specie laterum & angulorum.

Quia (ut alibi diximus) arcus & anguli eosdem habent sinus cum suis complementis ad duos rectos; hinc fit ut, invento per trianguli analysim sinu aliquo, dubitari possit, qualis, ipsi arcus vel angulus correspondeat: *v. g.* invento sinu 87 gr, dubitari potest utrùm angulus illi debitus sit 87 gr. an verò 93 gr. prioris complementum ad duos rectos. Ut huic incommodo occurratur, propositiones aliquas subjiciam, quæ ad determinandam laterum & angulorum speciem pro regulis erunt.

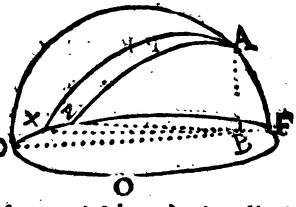
Consultius forsè erit ut caput hoc à Tyronibus initio prætermittatur, quia laterum & angulorum species plerumque potest aliunde determinari.

PROPOSITIO I.

Si ex puncto (A) quod non est polus circuli (DOE)

ejus circumferentiam ducantur circulorum maximorum arcus quotcunque (AD, AX, AZ;) *arcus* (AD) *per polum transiens eſt maximus: & reliquis ille major qui maximo vicinior.*

Demonſtr. Quia arcus DAF tranſit per polum circuli DOFZ, (*per 6. c. 1. h*) illi inſiſtit ad angulos rectos: ergo planum DAF (*per 4. def. c. 1. h*) eſt rectum plano DOFZ: ergo ſi ducatur AB perpendicularis ad ſectionem communem DF, erit AB (*per 4 def l. 11.*) recta plano DOFZ; adeoque (*per 3. def. l. 11.*) etiam lineis BD, BX, BZ: ergo triangula ABD, ABX, ABZ (ſubintelligi debent in hac figura chordæ arcuum AD, AX, AZ, quæ ad vitandam confuſionem non exprimuntur) ſunt rectangula in B: ergo cùm latus AB ſit commune, & BD per centrum tranſiens ſit (*per 7. l. 3.*) majus quàm BX, & BX quàm BZ; baſis AD (*per 5. Corol. p. 47. l. 1.*) major eſt baſi AX, & AX major baſi AZ: ergo (*per 26. l. 3.*) arcus AD major eſt arcu AX, & AX major arcu AZ.



PROPOSITIO II.

In triangulo ſpharico rectangulo, quodvis latus circa angulum rectum eſt ejuſdem ſpeciei cum angulo oppoſito.

Hoc eſt, ſi latus aliquod BX circa angulum rectum B ſit quadrans, angulus oppoſitus BAX erit rectus: ſi ſit majus minuſve quadrante, angulus erit ſimiliter major minorve recto: & è converſo.

Demonſtr. 1.° Quia angulus B eſt rectus, & BX quadrans; eſt X (*per 5. & 6. c. 1. h*) polus circuli BA. Ergo quia AX tranſit per polum, angulus BAX (*per 6. c. 1. h*) eſt rectus.

2.° Si ſit BF minus quadrante, baſis AF cadet inter X & B: ergo angulus BAF erit minor recto BAX.

3.o Si fit BZ majus quadrante, basis AZ cadet ultra X ergo angulus BAZ est major recto BAX.

Conversa patet; nam si anguli differrent specie à lateribus oppositis, latera non essent ejusdem speciei cum angulis oppositis.

PROPOSITIO III.

Si latera circa angulum rectum, vel anguli ad basim, sint ejusdem speciei, & neque quadrantes, neque recti, basis est minor: si sint diversa, est major quadrante.

Demonstr. I. Pars. Sint 1.o BA, BF minora quadrante: ergo (*per 5. c. 1. h*) A non est polus circuli BX: ergo (*per 1. h*) basis AF est minor quadrante AX.

2.o Sint circa C angulum rectum, CA, CF majora quadrante: A (*per 5. c. 1. h*) non erit polus circuli CXB: ergo (*per 1. h*) basis AF est minor quadrante AX, nam magis distat ab AC transeunte per polum.

2. *Pars.* Sit latus CA majus, & CZ minus quadrante. Quia CA est majus, A non est polus circuli CXB: & quia CZ est minus quadrante, basis AZ cadet inter C & polum X: ergo (*per 1. h*) est major quadrante AX.

Quod dictum de lateribus; verum est de angulis ad basim, quia sunt ejusdem speciei cum lateribus oppositis.

PROPOSITIO IV.

Si basis sit major quadrante, latera sunt diversa speciei: si minor, sunt ejusdem.

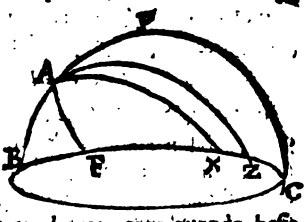
Patet ex præced: nam si latera essent ejusdem speciei, basis esset minor: ergo quando est major quadrante, sunt speciei diversæ. Similiter si essent diversæ speciei, basis esset major: ergo quando basis est minor, sunt ejusdem.

PROPOSITIO V.

Si latus aliquod (BX) circa angulum rectum (B) sit quadrans, etiam basis (AX) est quadrans: & è converso.

Demonstr. I. Pars. Quia angulus B est rectus, & BX quadrans, punctum X (*per 5. & 6. c. 1. h*) est polus circuli BC: ergo

ergo (*per. 5. c. 1. h*) basis AX est quadrans.



2. *Pars.* Si neutrum latus sit quadrans, sunt vel singula majora, vel singula minora, vel unum majus, alterum minus quadrante: sed in primo & secundo casu basis (*per 3. h*) est minor; & in tertio est major quadrante: ergo quando basis est quadrans, aliquod aliud latus est quadrans.

Corollarium.

In omni triangulo rectangulo vel duo tantùm latera majora sunt quadrante, vel nullum est majus; quòd enim non possint esse tria singula quadrante majora, statim ostendam. Unde quotescunque in triangulo rectangulo datur unum latus majus quadrante, produc latera duo majora donec fiant semicirculi, & habebis (*per. 2. Corol. p. 4. c. 1. h*) novum triangulum rectangulum, cujus singula latera erunt quadrante minora.

Propositiones explicatæ sufficiunt ad determinandam laterum & angulorum speciem, sextos casus excipias. Primus est quando vel duo anguli sunt recti, vel duo latera quadrantes; nam ex his nil potest de tertio angulo vel latere determinari. Secundus habetur, quando solùm cognoscitur latus aliquod circa angulum rectum, cum angulo ipsi opposito; nam possunt hæc esse ejusdem speciei in triangulis diversis, ut patebit consideranti triangula ACZ, ACX, ACF, item ABF, ABX, ABZ.

Quomodò & quatenùs hæc possint applicari triangulis obliquangulis postea ostendam.

PROPOSITIO VI.

Si in triangulo sphærico Isosceli (ABC) *latera æqualia* (AB, AC) *sint quadrantes, anguli* (B & C) *ad basim sunt recti; si sint majora, anguli sunt obtusi; si minora, sunt acuti.*

Demonstr. Quia latera AB, AC æquantur semicirculo, anguli B & C (*per 13. c. 1. h*) simul æquant duos rectos: ergo cùm (*per 11. c. 1. h*) sint æquales, ambo sunt recti.

Eodem modo (*per* 11. & 13. *c.* 1. *h*) demonstrantur secunda & tertia pars propositionis.

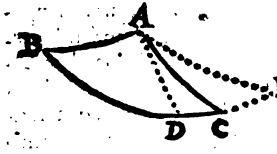
Corollarium.

Eodem ferè modo (*per conversas* 11 & 13. *c.* 1. *h*) demonstres, quòd si anguli ad basim sint ambo recti, latera AB, AC sint quadrantes: item quòd si anguli isti sint inter se æquales & simul majores duobus rectis, latera opposita AB, AC sint singula quadrante majora: minora verò, si anguli æquales B & C sint acuti.

LEMMA.

Si anguli (B, C) *ad basim sint obliqui & ejusdem speciei, perpendiculum ab alio angulo* (A) *ductum cadet intra: aliàs cadet extra triangulum.*

Demonstr. I. Pars. Quia si caderet extra, ad E *v. g.* anguli ABE, ACE (*per* 2. *h*) convenirent specie cum latere opposito AE; adeoque & inter se. Sed (*ex suppos.*) ABE, ACB sunt ejusdem speciei: ergo ACB, ACE sunt ejusdem speciei; quod est absurdum, cùm ACB ponatur obliquus, & ACE sit (*per* 2. *c.* 1. *h*) ejus complementum ad duos rectos.



2. *Pars.* Quia si perpendiculum AD cadat intra triangulum, anguli B & ACB erunt (*per* 2. *h*) ejusdem speciei cum latere opposito AD: ergo non different specie inter se.

PROPOSITIO VII.

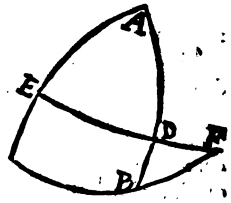
Si alicujus trianguli sphærici (ABC) *omnes anguli sint acuti, singula latera sunt minora quadrante.*

Demonstr. Ab angulo A demittatur perpendiculum AD. Quia anguli B, C sunt (*ex suppos.*) acuti, adeoque ejusdem speciei, AD cadet intra triangulum, & erunt in triangulo ADC ad basim AC anguli C & DAC ambo acuti: ergo (*per* 3. *h*) basis AC est minor quadrante. Si ab angulis B & C duxeris perpendi-

tula, idem eodem modo demonstres de lateribus AB, BC.

PROPOSITIO VIII.

Si trianguli sphærici (ABC) *singula latera sint quadrante majora, omnes anguli sunt obtusi.*



Demonstr. Si omnes anguli essent acuti, omnia latera (*per 7 h*) essent minora quadrante: ergo aliquis est vel rectus, vel obtusus. Sit ille angulus A. Centro A, intervallo quadrantis, describe arcum ED concurrentem in F cum CB producto. Erit A (*per 5. c. 1. h*) polus circuli ED, & arcus AE, AD quadrantes: ergo angulus AED, adeoque (*per 2. c. 1. h*) & CEF est rectus. Item ED est (*per 3. c. 1. h*) mensura anguli A vel recti, vel obtusi. Ergo ED est ad minimum quadrans, adeoque EF est major quadrante. Sed etiam arcus CB (*ex suppos.*) adeoque & CF est major quadrante: ergo CF, EF simul excedunt semicirculum: ergo (*per 13. c. 1. h*) anguli ECF, CEF simul excedunt duos rectos. Sed (*ex dictis*) CEF est rectus: ergo ECF seu angulus C est obtusus. Eodem modo, si produxeris DE, BC donec concurrant versùs E & C, ostendas angulum B obtusum: idemque similiter ostenderur de angulo A.

Corollaria.

1. *Eodem ferè modo ostendas omnes angulos esse obtusos, si sint duo latera* AC, AB *majora quadrante, & tertium* CB *quadrans.*

2. *Si in triangulo aliquo duo latera* BD, CD *sint minora quadrante, & tertium* BC *majus; angulus maximo lateri oppositus, est obtusus, & duo reliqui acuti.*

Producantur DB, DC, donec concurrunt in A, erunt (*per 2. Corol. p. 4. c. 1. h*) trianguli ABC latera omnia majora quadrante, adeoque omnes anguli obtusi. Sed anguli A & D sunt æquales: ergo D est obtusus. Anguli verò DBC, DCB sunt (*per 2. c. 1. h*)

complementa obtusorum ACB, ABC, adeoque acuti.

3. *Si in triangulo* ABC, *duo anguli quivis* B & C *sint obtusi, aliqua duo latera sunt majora quadrante.*

Nam si tertius A sit acutus, trianguli oppositi DBC anguli omnes erunt acuti: ergo (*per* 7. *h*) latera DB, DC sunt minora, adeoque AB, AC sunt majora quadrante.

Si verò omnes anguli sint obtusi; quia A & B sunt majores duobus rectis: arcus AC, BC sunt simul (*per* 13. *c.* 1. *h*) majores semicirculo: ergo aliquis eorum est major quadrante. Eodem modo ostendam AC & AB, item BC & AB excedere semicirculum: ergo aliqua duo latera singula excedunt quadrantem.

4. *Si in triangulo obliquangulo sint duo latera minora quadrante, duo anguli sunt acuti.*

Nam si duo essent obtusi, duo latera (*per præced.*) essent majora quadrante.

5. *Si in triangulo obliquangulo aliquod unum latus sit majus quadrante, aliquis angulus est obtusus.*

Nam si omnes anguli essent acuti, omnia latera (*per* 7. *h*) essent minora quadrante.

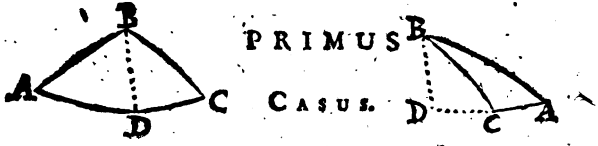
6. *Si nullus angulus sit acutus, latus aliquod est minus quadrante.*

Nam si omnia essent majora, omnes anguli (*per* 8. *h*) essent obtusi. Si omnia essent quadrantes, omnes anguli (*per* 3. *c.* 1. *h*) essent recti. Si duo essent quadrantes & tertium latus majus quadrante; anguli quadrantibus oppositi (*per* 11. & 13. *c.* 1. *h*) essent recti, angulus verò majori lateri oppositus (*per* 3. *c.* 1. *h*) major recto. Demum si duo latera essent quadrante majora & unum quadrans, omnes anguli (*per* 1. *Coroll. h*) essent obtusi.

PROPOSITIO IX.

In triangulis obliquangulis laterum & angulorum species determinare.

Si in triangulo proposito dentur tria latera, vel tres anguli, præsens difficultas locum non habet; nam (*per* 4. & 5. *c.* 3. *h*) invenitur semissis anguli verticalis, quæ semper est (*per* 10. *c.* 1. *h*) minor recto. Tota igitur difficultas reducitur ad alios quatuor casus quos hîc breviter explicabimus.



Quando dantur duo latera (BA, AC) *& angulus contentus* (A,) *reliquorum ſpeciem invenire.*

Ducatur perpendiculum BD. In triangulo rectangulo BAD, ex angulo A (*per* 2. *h*) cognoſcitur ſpecies lateris BD. Ex BA baſi & latere BD (*per* 4. *h*) habetur ſpecies lateris AD. Inveniatur (*per* 3. *c.* 2. *h*) latus AD, & notum erit latus DC. Ergo in triangulo BDC habetur ſpecies laterum BD, DC; adeoque (*per* 2. *h*) & angulorum C & DBC, & [*per* 3. *h*] baſeos BC.

SECUNDUS CASUS.

Quando dantur duo anguli [A & B] *cum latere interjecto* [BA] *reliquorum ſpeciem determinare.*

Ab aliquo angulo noto [B] demittatur perpendiculum BD. In triangulo rectangulo BAD, ex angulo A [*per* 2. *h*] ſcitur ſpecies lateris BD. Ex ſpecie lateris BD & baſeos BA [*per* 4. *h*] cognoſcitur ſpecies lateris AD. Invento [*per* 3. *c.* 2. *h*] angulo ABD, habetur angulus CBD, adeoque [*per* 2. *h*] ſpecies lateris DC. Ergo in triangulo DBC ſcitur ſpecies laterum BD, DC; adeoque [*per* 2. *h*] angulorum C & CBD, [*per* 4. *h*] baſeos BC.

TERTIUS & QUARTUS CASUS.

Notis duobus lateribus cum angulo eorum uni oppoſito; vel duobus angulis cum latere quod opponitur alicui angulo noto, cæterorum ſpeciem determinare.

Pro his duobus caſibus non eſt aliqua regula ſeu methodus generalis, per quam reliquorum arcuum & angulorum ſpecies poſſit univerſaliter determinari. In caſibus particularibus ad hoc inſervient *Corollaria poſt Prop. præced. & cap.* I *Prop.* 9. 10. 11. 12. 13. quæ Lector conſulat.

CAPUT V.

Selecta quædam Problemata per Trigonometriam Sphæricam resolvuntur.

Ut appareat usus & utilitas eorum quæ hactenus explicata sunt, Problemata nonnulla selegi, pro regulis datis exemplorum loco, & in aliis Matheseos partibus usui futura; quæ, ut faciliùs percipiantur, aliqua terminorum explicatio est præmittenda.

DEFINITIONES.

1. *Axis* mundi est diameter circa quam sphæra mundi circumagitur. *Poli* verò sunt hujus axis duo puncta extrema.

2. *Æquator* est circulus maximus, cujus poli sunt iidem cum polis mundi.

3. *Ecliptica* est circulus maximus, quem Sol unius anni spatio percurrit, obliquè secat Æquatorem ad angulum 23 gr. 30'. Sectionis puncta vocantur *Æquinoctialia*, quia Sol eò perveniens, dies noctibus adæquat. Puncta verò Ecliptica ab Æquatore remotissima dicuntur *Solstitialia*, quia Sol ad ea pertingens videtur quasi stare & in punctis illis consistere.

4. *Horizon Rationalis* est circulus maximus, cujus unus polus est punctum cæli quod cujusvis vertici directè imminet, & vocatur *Zenith*; alter, qui dicitur *Nadir*, est punctum priori directè oppositum. *Horizon sensibilis* vel *Physicus* est circulus priori parallelus, qui undique visum terminat.

5. *Circuli verticales* sunt circuli maximi per Zenith & Nadir ducti. Horum unus qui per mundi polos transit, *Meridianus* appellatur: alter verò *Verticalis Primarius*, qui transit per puncta veri Orientis & Occidentis, id est, per illa puncta in quibus Æquator Horizontem secat.

Circuli verticales vocantur etiam *circuli altitudinis*, quia *stellæ* v. g. *altitudo* est arcus circuli verticalis inter Horizontem & centrum sideris interceptus. Sideris *Profunditas* vel *Demersio* est arcus circuli verticalis inter Horizontem & centrum sideris infra Horizontem existentis. Unde etiam Poli & Æquatoris altitudo, est arcus Meridiani, inter Polum vel Æquatorem & Horizontem interceptus.

6. *Azimuth* ſideris eſt arcus Horizontis interceptus inter Meridianum & circulum verticalem per centrum ſideris tranſeuntem.

7. *Amplitudo Ortiva* vel *Occidua* ſideris eſt arcus Horizontis inter ſidus oriens vel occidens & verum Orientem vel Occidentem ſeu Æquatorem.

8. *Circuli Declinationis* ſunt circuli maximi per polos mundi tranſeuntes: eorum arcus interceptus inter Æquatorem & ſideris alicujus centrum, dicitur illius *declinatio*. Duodecim declinationis circuli dividentes Æquatorem in 24. partes æquales, vocantur *Circuli Horarii*: Duo verò tranſeuntes per puncta Æquinoctialia & Solſtitialia, vocantur *Coluri*.

9. *Aſcenſio* ſideris vel alicujus puncti Eclipticæ, eſt arcus Æquatoris inter ſectionem vernam, ſeu initium Arietis verſus ortum; & illud Æquatoris punctum, quod unà cum illo ſidere ad Horizontem pertingit.

Aſcenſio recta eſt quæ fit in ſphæra recta, ubi ſcil: Æquator Horizontem ſecat ad angulos rectos.

Aſcenſio obliqua eſt illa quæ fit in ſphæra obliqua, ubi Æquator obliquè ſecat Horizontem.

Differentia aſcenſionalis eſt in Æquatore diſtantia puncti quod oritur unà cum ſidere aut puncto Eclipticæ in ſphæra recta, à puncto quod unà cum illis oritur in ſphæra obliqua.

Ex dictis de Aſcenſione, facilè intelligetur quid ſit ſideris vel puncti Eclipticæ *Deſcenſio recta* vel *obliqua*, & *differentia deſcenſionalis*.

10. *Circuli longitudinis & latitudinis ſiderum* ſunt circuli maximi per polos Eclipticæ & ipſa ſidera tranſeuntes. *Longitudo ſideris* eſt arcus Eclipticæ inter initium Arietis & circulum longitudinis per ſidus tranſeuntem. *Latitudo* eſt arcus circuli latitudinis inter ſidus & Eclipticam contentus. Si ſidus declinet ad Auſtrum, dicitur latitudo *Auſtralis*; ſi ad Septentrionem, *Borealis*.

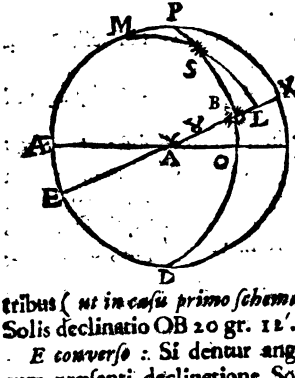
11. *Circuli longitudinis & latitudinis Geographica* ſunt ipſi Meridiani. Unde *Latitudo* loci in ſphæra terreſtri eſt arcus Meridiani inter locum & Æquatorem interceptus. *Longitudo* eſt arcus Æquatoris inter Meridianum loci & primum Meridianum contentus.

Nota non omnes Geographos in eodem loco constituere primum Meridianum. Angli & Belgæ à Monte Teneriffa, Galli à parte occidentali Insulæ de *Ferro*, alii ab aliis locis longitudinum initium sumunt. Unde contingit in designanda locorum longitudine magna inter Authores discrepantia. Observari igitur debet inter legendum, quod longitudinis initium unusquisque Author statuerit.

PROBLEMA I.

Cognitâ maximâ declinatione Solis (quæ est gr. 23. 30') & ejus distantiâ à proximo puncto Æquinoctiali, invenire præsentem declinationem: & è converso, datâ præsenti Solis declinatione, & declinatione maximâ, invenire ejus locum in Ecliptica.

Sit in Ecliptica EX locus Solis B, distans 60 gr. ab initio Arietis A. In triangulo BOA, angulus O (*per 6. c. 1. h*) est rectus, nam declinationis circulus PBD (*per 8. def. h*) transit per P, D Polos Æquatoris ÆR; angulus A (*per 3. c. 1. h*) eodem gradus continet quos XR maxima Solis declinatio, viz. 23. gr. 30'. Et latus AB (*ex suppos.*) est 60. gr. Ex his tribus (*ut in casu primo schematis c. 2. h*) invenitur præsens Solis declinatio OB 20 gr. 12'.



E converso: Si dentur anguli O rectus, & A 23 gr. 30' cum præsenti declinatione Solis OB 20 gr. 12'. Ex his tribus (*ut in casu 7. ejusdem schem.*) invenitur Solis in Ecliptica locus B distare 60 gr. ab initio Arietis A, id est, arcum BA esse 60 gr.

PROBLEMA II.

Iisdem cognitis invenire ascensionem rectam Solis aut alicujus puncti Ecliptica: & è converso, his datis invenire Solis declinationem.

Sit B locus Solis vel puncti in Ecliptica, cujus quæritur ascensio

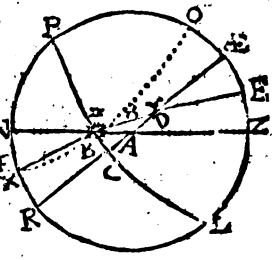
ascensio recta AO. In triangulo AOB angulus A est (*ut prius*) 23 gr. 30′: angulus O est rectus, quia (*per def. 9. h*) in sphæra recta Æquator ÆR est rectus Horizonti PBD: latus verò AB supponitur notum, *v. g.* 60 gr. quale est quando Sol est in initio Geminorum. Ex his tribus (*ut in 2. casu 1. schem.*) eruitur latus AO 57 gr. 48′. Ascensio recta puncti B.

Et è converso, datâ ascensione rectâ AO, cum angulis O & A, invenitur (*ut in 4. casu 1. schem.*) BO Solis declinatio.

PROBLEMA III.

Datâ Poli altitudine cum declinatione Solis invenire differentiam ascensionalem, & consequenter ascensionem obliquam: & è converso, datâ differentiâ ascensionali invenire Solis declinationem.

Sit in EX Ecliptica, B locus Solis orientis in Horizonte obliquo ZN; sit etiam Æquator ÆR ejusque polus P. Horizon rectus & circulus declinationis Solis sit PCL; erit ascensio recta DC, ascensio obliqua DA, differentia ascensionalis AC. In triangulo ABC, latus BC est declinatio Solis data, *v. g.* gr. 20. 12′: angulus C (*per 6. c. 1. h*) est rectus: angulus A æqualis est arcui NR distantiæ Horizontis ZN ab Æquatore ÆR: quæ distantia est complementum altitudinis poli, ut paschic intuenti sphæram Armillarem. Cùm igitur hîc Leodii altitudo poli sit 50 gr. 40′, erit ejus complementum, adeoque & angulus A 39 gr. 20′. Ex his tribus (*ut in 5. casu 1. schem.*) invenitur latus AC differentia ascensionalis 26 gr. 41′. Hanc subtrahe ex DC ascensione rectâ 57 gr. 48′. priùs inventâ, residuum 31 gr. 7′. erit ascensio obliqua DA.



E converso. In triangulo ABC notâ differentiâ ascensionali AC, & angulis A & C, invenitur (*ut in 4. casu 1. schem.*) declinatio Solis BC.

Nota, quando Sol Borealis est, differentiam ascensionalem

subtrahendam esse ascensioni rectæ, ut habeatur ascensio obliqua: addendam verò, ut habeatur descensio obliqua. Quando autem Sol Australis est, invenitur ascensio obliqua addendo, & descensio obliqua subtrahendo ascensioni rectæ differentiam ascensionalem.

Corollarium.

Notâ Solis differentiâ ascensionali determinare horam Ortûs vel Occasûs Solaris.

Quamvis Solis motus reverâ spiralis sit; Astronomi tamen, calculi commoditate ducti, imaginantur Solem motu diurno describere circulum vel in æquatore vel æquatori parallelum; cujus portio illa quæ supra Horizontem extat, vocatur *Arcus diurnus*, estque in Æquinoctiis semicirculus æquatoris, extra Æquinoctia aliquando major, interdum minor semicirculo. Unde sequitur arcum semidiurnum nunc excedere, nunc deficere à quadrante. Ut igitur pro die constituto inquiratur ortus Solis, inveniri debet 1.o quantus sit arcus semidiurnus, quem Sol tunc describit. 2.o spatium temporis isti Solis motui correspondens.

Sit PRÆ Circulus Meridianus, OF arcus diurnus; reliqua constructio ut in *Probl.* 3. Jam quia Solis existentis in B ascensio recta est (*ex dictis*) DC, & ascensio obliqua DA; erit differentia ascensionalis CA: & quia Sol oritur in B, erit BO arcus semidiurnus, qui similis est æquatoris arcui CÆ. Sed differentia inter arcum CÆ & quadrantem AÆ est ascensionalis differentia CA. Cùm igitur, Sole existente in signis borealibus, arcus semidiurni sint in Hæmisphærio boreali quadrante majores; minores verò, dum Sol est in Australibus; sequitur quòd differentia ascensionalis, quando Sol Borealis est, addi debeat quadranti, ut habeatur arcus semidiurnus; subtrahi verò, quando est Australis. Unde addendo quadranti differentiam ascensionalem 26. gr. 41'. suprà (*Probl.* 3.) inventam, habebitur arcus semidiurnus 116. gr. 41'. cùm Sol est in initio Geminorum.

Deinde, quia singulis 24 horis æquator & circuli omnes ei paralleli toti transeunt per Meridianum motu æquabili; sequitur, quòd 6 horarum spatio per Meridianum transeat

quadrans unus tum æquatoris, tum cujuscunque circuli paralleli; unde differentia ascensionalis in tempus conversa, & addita vel subtracta (prout Solis locus exigit) 6 horis, dabit tempus correspondens arcui semidiurno.

Differentiam verò ascensionalem, vel quemcunque æquatoris arcum in tempus convertere, non est aliud quàm determinare, quanto tempore arcus ille per Meridianum transeat. Cùm igitur horis singulis una vigesima quarta æquatoris pars seu 15 gradus per Meridianum transeant; invenietur tempus cuicunque arcui correspondens, dividendo ejus gradus & minuta per 15 : vel, faciliùs, ea multiplicando per 4, mutatis in producto gradibus in minuta, & minutis primis in secunda. Sic ascensionalis differentia 26 gr. 41′ multiplicata per 4 dar, signis mutatis, 1 h. 46′. 44″. seu Hor. 1. 46′. 44″.

G.	
26 - -	41′
	4
106′ - -	44″ *id est*
1 h - - 46 - -	44

Operationis ratio obvia est; nam Gradus deprimere ad Minuta, & prima mutare in secunda, est productum dividere per 60. Multiplicare verò per 4, & productum dividere per 60, non est aliud quàm dividere per 15, ut requiritur.

Tempus differentiæ ascensionali correspondens viz: 1 h 46′. 44″. (quia Gemini sunt signum boreale) additum 6 horis, dat 7 h 46′. 44″. quibus arcus CÆ (cui similis est BO) totus Meridianum transit, seu tempus semidiurnum. Unde Sol in initio Geminorum oritur Leodii, horis 7. 46′. 44″ ante meridiem, id est, horâ 4. 13′. 16″; occidit autem horâ 7. 46′. 44″.

PROBLEMA IV.

Datâ Poli altitudine cum declinatione Solis, vel differentiâ ascensionali, invenire amplitudinem ortivam vel occiduam.

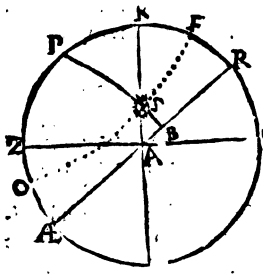
Positis omnibus ut in *Probl.* 3. In triangulo ABC, angulus C est rectus, angulus A 39 gr. 20′. & latus BC 20 gr. 12′. Ex his (*ut in* 7. *casu* 1. *schem.*) invenitur amplitudo ortiva vel occidua AB 33 gr. 1′.

Similiter datâ in eodem triangulo differentiâ ascensionali AC; ex AC & angulis A, C, invenitur (*ut in* 8. *casu* 1. *schem.*) basis AB.

Et è converſo, notâ baſi AB ſeu amplitudine ortivâ cum angulis A, C, invenitur (*ut in* 1. *caſu* 1. *ſchem.*) declinatio Solis BC, & (*ut in* 2. *caſu*) differentia aſcenſionalis AC.

PROBLEMA V.

Cognitâ altitudine Poli, & declinatione Solis, determinare tempus quo Sol perveniet ad Circulum veri Orientis vel Occidentis, ſeu ad verticalem primarium: & è converſo.



Sit PÆR Meridianus. ZN Horizon ÆR Æquator, eique parallelus OF. K Zenith. PS circulus declinationis & ſimul horarius tranſiens per centrum Solis exiſtentis in S puncto verticalis primarii KSA. His poſitis, in triangulo ABS, angulus B (*per* 6. *c.* 1. *h*) eſt rectus anguli SAB menſura eſt KR diſtantia Zenith ab æquatore, quæ æqualis eſt altitudini poli; adeoque Leodii angulus SAB eſt 50 gr. 40′. Arcus SB eſt declinatio Solis, quæ in initio Geminorum eſt 20 gr. 12′. Ex his (*ut in* 5. *caſu* 1. *ſchem.*) invenitur arcus AB 17 gr. 33′. Unde ejus complementum BR diſtantia Æquatoria Solis à Meridiano PKR, eſt gr. 72. 27′. quæ ad tempus reducta (*ut in Corol. Probl.* 3.) dat 4. horas. 49′. 48″. Sol igitur in initio Geminorum attingit circulum veri Orientis horâ 7. 10′. 12″. ante, & veri Occidentis horâ 4. 49′. 48″ poſt meridiem.

E converſo. Si detur hora quâ Sol appellit ad verticalem primarium KSA, ejus diſtantia à meridie converſa (*ut in Corol. Probl* 3.) in gradus æquatoris dabit arcum BR, adeoque & ejus complementum BA. Ex quo & angulis B, BAS, invenitur (*ut in* 4. *caſu* 1*e ſchem.*) SB declinatio Solis.

PROBLEMA VI.

Cognitâ Solis declinatione & Poli altitudine, altitudinem Solis quâvis diei horâ invenire, v. g. *horâ octavâ.*

Fiat eadem conſtructio, quæ *in Probl.* 5. In triangulo

PKS, arcus PK est complementum altitudinis Poli, adeoque gr 39. 20'. Arcus PS est complementum declinationis solaris SB, quæ si ponamus Solem in initio Tauri *v. g.* est 11 gr. 30'. unde PS habebit 78 gr. 30'. Angulus verò SPK (*per 3. c. 1. h*) æqualis est arcui æquatoris BR, qui mensurat distantiam æquatoriam Solis à Meridiano PÆR; & invenitur dicendo, ut 1 hora ad 15 gradus: sic tempus inter horam constitutam & meridiem, viz: 4 horæ, ad aliud. Prodeunt 60 gr, adeoque tantus est angulus SPK. Ex hoc angulo & lateribus PK 39 gr. 20', PS 78 gr. 30'. invenitur (*ut in 2. casu 2. sphem.*) latus KS gr 62. 19'. cujus complementum SA 27 gr. 41'. est altitudo Solis quæsita.

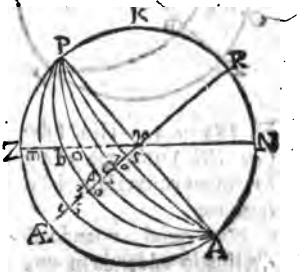
PROBLEMA VII.

Cognitâ Poli altitudine cum Solis declinatione & altitudine, invenire horam diei vel Solis Azimuth.

Detur altitudo Solis (*vide Fig. Probl. 5.*) SA *v. g.* 27. gr. 41'. erit ejus complementum KS 62. gr. 19'. Detur etiam declinatio Solis SB, *v. g.* 11 gr. 30'. erit ejus complementum PS 78. gr. 30'. Sit altitudo Poli ZP 50 gr. 40'. erit ejus complementum PK 39 gr. 20'. Ergo in triangulo PSK nota sunt omnia latera. Ex quibus (*ut in Prop. 4. c. 3. h*) invenitur angulus P 60 gr. qui in tempus mutati sunt 4 horas. Ex iisdem inveniri potest angulus PKS; id est (*per 3. c. 1. h*) arcus AZ Solis Azimuth.

PROBLEMA VIII.

Cognitâ Poli altitudine invenire arcus Horizontis qui inter Meridianum & alios Horarios circulos intercipiuntur.



Sit PÆR Meridianus. ÆR Æquator. ZN Horizon. P Polus. K Zenith. PZA, PMA, PBA &c. circuli Horarii. In triangulis ZPM, ZPB &c. quia Meridianus ZP transit per Zenith K polum Horizontis, angulus Z (*per 6. c. 1. h*) est rectus. Anguli verò ad P noti-

sunt, nam (*per 6. c. 1. h*) sunt ejusdem mensuræ cum arcubus æquatoriis Æ 15. Æ 30. &c. Denique latus ZP est altitudo Poli quæ supponitur nota. Ex his tribus (*ut in 4. casu 1, schem.*) inveniuntur arcus ZM, ZB, ZO &c. quæsiti.

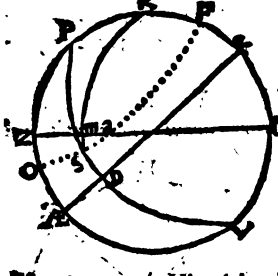
PROBLEMA IX.

Datâ Poli altitudine & declinatione Solis, invenire durationem Crepusculi.

Docent Astronomi plerique Crepusculum manè inchoari, & vesperi desinere, quando Sol deprimitur 18 gradibus infra Horizontem. Quærimus quantitatem temporis interjecti inter initium Crepusculi matutini & ortum Solis, sive inter ejusdem occasum & finem Crepusculi vespertini.

Sit PRÆ Meridianus. ZN Horizon. K Zenith. Circulus verticalis KMS. Polus P. Circulus declinationis & simul Horarius PSL. Æquator ÆR, eique parallelus circulus OF. Solis locus S in initio Geminorum.

In triangulo PSK latus PK est complementum altitudinis Poli, adeoque hîc Leodii est 39. gr. 20′. Latus PS habetur subtrahendo ex quadrante PD borealem Solis declinationem DS, quæcùm sit in initio Geminorum 20. gr. 12′. latus PS est 69. gr. 48′. Latus KS habetur addendo Solis profunditatem SM, seu 18 gr. quadranti KM; adeoque KS est 108 gr. Notis sic tribus lateribus PK, PS, KS invenitur (*per 4. c. 3. h*) angulus P, id est (*per 3. c. 1. h*) arcus RD seu FS 165 gr. 34′. Hinc subtrahe arcum semidiurnum FA, qui (*ex dictis*) tunc est 116 gr. 41′. & residuum SA 48 gr. 53′. in tempus conversum dabit 3 horas. 15′. 32″. pro Crepusculo quæsito.

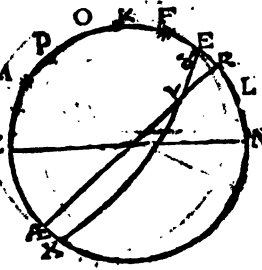


Nota quòd, quando Sol est in signis Australibus, ejus declinatio addenda sit quadranti PD.

PROBLEMA X.

Alicujus ſtellæ fixæ declinationem, aſcenſionem rectam & longitudinem & latitudinem inveſtigare.

1.o Stellæ propoſitæ ad Meridianum ZKN appulſæ capiatur altitudo quadrante exquiſito. Si ſtella ſit in quadrante boreali KZ, & infra Polum P, *v. g.* in A; altitudini inventæ AZ addatur arcus ZÆ profunditas æquatoris ÆR infra Horizontem ZN, & notus erit arcus AÆ declinatio quæſita. Si ſit inter Polum P & Zenith K, *v. g.* in O, ejus à Zenith diſtantia OK addita Poli altitudini PZ ſeu KR, dabit OR declinationem quæſitam. Si ſtella ſit in quadrante Auſtrali, eandemque habeat altitudinem Meridianam cum æquatore, nullam habet declinationem. Si verò altior ſit, *v. g.* in F; ejus altitudini FN ſubtracta æquatoris altitudo RN relinquet declinationem FR quæſitam. Si denique ſtella ſit inter æquatorem & Horizontem, *v. g.* in L, ejus declinatio LR eſt Auſtralis, & habetur ſubtrahendo ſtellæ altitudinem LN. Ex æquatoris altitudine RN.

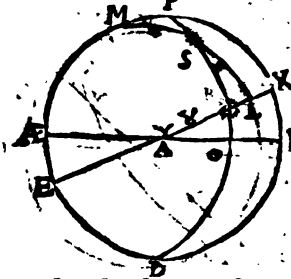


2.o Ut inveniatur aſcenſio recta alicujus ſtellæ, menſuretur accuratè (numerando Penduli vibrationes, vel aliter) tempus inter Meridiem proximè præcedentem, & ſtellæ appulſum ad Meridianum, qui diligenter debet obſervari. Tempus verò inventum mutetur in gradus æquatoris (quod ſit temporis inventi horas & minuta multiplicando per 15) quibus addatur aſcenſio Solis recta pro Meridie præcedenti (*per Probl. 3. h*) computata. Eritque eorum ſumma aſcenſio ſtellæ recta, quæ quæritur.

Ex. gr. Sole exiſtente in initio Geminorum, adeoque habente (*per Probl. 2.*) aſcenſionem rectam 57 gr. 48′; obſervatus fuerit anno 1704. noctis horâ 10. 10′. 59″. 56‴. appulſus ſtellæ, *v. g.* Arcturi ad Meridianum. Tempus hoc converſum in æquatoris gradus, dat 152 gr. 44′. 59″. 2‴.

quibus si addatur Solis ascensio recta; summa 210 gr. 32′. 59″. erit Arcturi ascensio recta quæsita.

3.° Ut exploretur stellæ alicujus longitudo vel latitudo, sit ÆR Æquator. EX Ecliptica. ÆPRD Colurus Solstitialis. PSD circulus declinationis stellæ existentis in S, cujus proinde declinatio est SO. Demum sit MSL circulus latitudinis ejusdem stellæ. His positis, erunt in triangulo MPS, latus PS complementum declinationis SO, quæ supponitur nota; latus MP 23. gr. 30′. distantia inter Polos æquatoris & Ecliptica; angulus verò MPS (*per* 3. *c.* 1. *b*) mensuratur ab æquatoris arcu ÆQ, qui habetur addendo vel subtrahendo quadranti stellæ propositæ ascensionem rectam, vel ejus excessum supra semicirculum. Quid autem horum fieri oporteat, non aliunde melius quàm ex ipsa sphæra constabit, in qua attentè considerari debet circulorum propositorum situs, & cum siderum cælorumque motu conferri. Ex his tribus (*ut in* 7. *casu* 2. *vel* 3. *schem.*) invenitur latus MS, complementum arcûs SL latitudinis quæsitæ: & (*ut in* 8. *casu* 2. *vel* 3. *schem.*) angulus SMP, quem (*per* 3. *c.* 1. *b*) mensurat arcus XL, qui (prout stellæ locus exigit) additus aut subtractus uni pluribusve circuli quadrantibus, dabit longitudinem quæsitam.



Nota 2.° stellas fixas circumagi motu proprio circa Polos Ecliptica, & annis singulis conficere 50 circiter secunda. Unde fit ut, quamvis earum latitudines sint semper eædem, mutentur tamen longitudines, declinationes, & ascensiones rectæ; quia mutatur earum situs respectu primi Mobilis & æquatoris. Stellarum aliquot nobiliorum longitudines, &c. exhibet Tabula sequens, à Ricciolo computatas pro anno 1700 completo, cum ascensionum & declinationum differentiis pro annis 100, quarum ope cuivis anno possunt facilè accommodari.

TABULA

TABULA

Longitudinum, Latitudinum, Ascensionum Rect. & Declinationum Stellarum cum Differentiis Ascension. & Declinat.

	Lucida Lyræ	Arcturus	Cor Leonis
	Gr.	Gr.	Gr.
Longitudo	281. 06′. 17″	200. 04′. 27″	145. 38′. 32″
Latitudo	61. 47 00	31. 00. 40″	0. 26. 20
Ascensio recta	276 39 32	210. 33. 02	148. 04. 15
Differ. Ascen.	0. 50. 00	1. 11. 00	1. 22. 30
Declinatio	38. 32. 16	20. 48. 02″	13 25. 16′
Differ. Declin.	A. 0. 04. 00	S. 0. 29. 30	S. 0. 28. 30
	Can. cum	Capella	Caput Andr.
	Gr.	Gr.	Gr.
Longitudo	111. 40′. 29″	77. 40′. 02″.	19. 09′. 03″
Latitudo	15. 57. 10	22. 51. 45	25. 42. 10
Ascensio recta	110. 54. 33	73. 35. 56	358. 14. 08
Differ. Ascen.	1. 20 00	1. 49. 00	1. 17. 00
Declinatio	5. 59. 12	45. 40. 00	27. 27. 26
Differ. Declin.	S. 0. 12. 00	A. 0. 10. 00.	A. 0. 34. 00

Ad Tabulæ hujus usum est diligenter observandum

1.° Quòd Fixarum longitudo augeatur annis singulis min. 50″. 40‴. quæ si multiplices per numerum annorum interjectum inter an. 1700 & tempus datum, productumque addas longitudini in Tabula inventæ; summa erit Fixæ longitudo ad tempus datum.

2.° Latitudines Fixarum nunquam mutantur.

3.° Si quæratur Stellæ alicujus Ascensio recta, fiat ut anni 100, ad differentiam Ascensionis in Tabula positam: sic anni inter an. 1700 & tempus datum, ad aliud. Quartus terminus additus Ascensioni in Tabula inventæ, dabit Ascensionem quæsitam. Eadem methodus tenenda est, quando quæritur declinatio. Quartus tamen terminus addi debet, vel subtrahi declinationi, quæ in Tabula habetur; prout vel litera A addendum, vel litera S subtrahendum indicat.

Nota 1.° Quòd si tempus, pro quo quæritur longitudo, vel

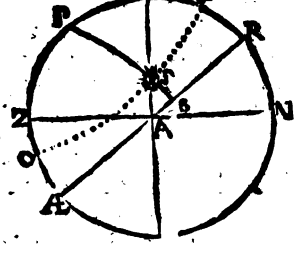
ascensio, vel declinatio stellæ, antecesserit annum 1700; adhibenda sit subtractio, ubi additione usi sumus, & è contrà.

Nota 2.o Boream versùs declinare stellas omnes in hac Tabula contentas: Caniculæ tamen latitudo est Australis; licèt declinatio sit Borealis.

PROBLEMA XI.

Ex dictis invenire noctis horam per stellas.

Sit constructio eadem quæ *Probl.* 5. & 6. sed S sit locus stellæ, Arcturi *v. g.* cujus inventa sit altitudo AS 50 gr. adeoque ejus complementum KS gr. 40. Deinde, quia ZP Poli altitudo est 50 gr. 40'; erit ejus complementum PK 39 gr. 20'. Denique latus PS habetur subtrahendo SB stellæ declinationem (vel eam addendo, si sit Australis) ex quadrante PB. Cùm igitur Arcturi declinatio sit Borealis, & (anno 1704.) 20. gr. 46'. 51". erit PS 69 gr. 13'. 9". Ex his tribus arcubus invenitur (*per* 4 *c.* 3. *b*) in triangulo PSK angulus P, adeoque (*per* 3. *c.* 1. *b*) arcus BR 33 gr. 56'. Hoc facto, ex Arcturi ascensione rectâ 210 gr. 33'. subtrahatur (si minor sit) Solis, existentis *ex. gr.* in initio Cancri, ascensio recta 90 gr. Residuum 120 gr. 33'. erit distantia Æquatoria Solis ab Arcturo; cui distantiæ si addatur BR 33 gr. 56' habebitur Solis à Meridiano distantia Æquatoria 154 gr. 29. quæ in tempus conversa dat 10 h. 17'. 56". noctis horam quæsitam.



Nota 1.o Quòd si ascensio recta Solis major sit ascensione rectâ stellæ, hæc debeat ex illa subtrahi, ut habeatur distantia Solis Æquatoria à stella proposita.

Nota 2.o Quòd si BR distantia stellæ à Meridiano sit (ut in hac Figura) Occidentalis, addi debeat arcus BR Solis distantiæ Æquatoriæ ab ipsa stella: sin Orientalis sit, debeat subtrahi.

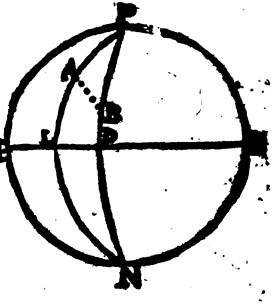
Ut ad minum semper haberentur declinationes & ascen-

ſones ſtellarum, quas utiliſſimi hujus Problematis ſolutio requirit; in Tabula præcedente ſelectæ ſunt ſtellæ in Hæmiſphærio noſtro conſpicuæ & ita diſperſæ, ut earum aliqua ſemper inſervire poſſit ad finem intentum.

PROBLEMA XII.

Datis Longitudine & Latitudine duarum Urbium vel Stellarum, invenire earum diſtantiam.

Sit primus Meridianus ÆPR. Æquator ÆR. Sit etiam A Urbs aliqua, Leodium *v. g.* ejuſque Meridianus PLN., Longitudo ÆL 28 gr. 38′. Latitudo AL 50 gr. 40′. Sit B Hieroſolyma, ejuſque Meridianus PDN, Longitudo ÆD 63 gr. 37′. Latitudo DB 32 gr. Hinc in triangulo APB cognoſcuntur latera AP, BP complementa latitudinum notarum AL, DB; unde AP eſt 39 gr. 20′. BP 58 gr. Angulus verò ABP menſuratur (*per* 3. *c.* 1. *h*) ab arcu LD longitudinum datarum differentiâ, quæ eſt 34 gr. 59′. Ex his tribus invenitur (*ut in* 1. *caſu* 2. *vel* 3. *ſchem.*) arcus AB 31 gr. 46′. Qui (methodo ſuprà traditâ) in Milliaria converſus, dat Milliaria Bononienſia 2047 & paſſus 816, ſeu Milliaria Horaria 682 & paſſus 1816.



Si A & B intelligantur eſſe duæ ſtellæ, & ÆR Ecliptica; eodem modo invenietur earum diſtantia.

Idem facilius præſtatur ope ſphæræ, ex qua ſi diſtantia inter loca propoſita circino excipiatur, & applicetur alicui circulo maximo in gradus diviſo, ſtatim innoteſcet quantus ſit arcus interceptus.

PROBLEMA XIII.

Cognitâ ſtella aſcenſione rectâ & declinatione, invenire ejus aſcenſionem obliquam, amplitudinem ortivam vel occiduam, & arcum diurnum vel nocturnum.

Sit PRÆ Meridianus. ÆR Æquator. B locus ſtellæ in Horizonte obliquo NZ. Horizon rectus & circulus declinationis

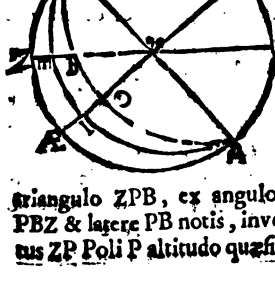
stellæ sit PCL. Erit stellæ ascensio recta DC; obliqua AD; differentia ascensionalis CA; amplitudo ortiva BA; stellæ declinatio BC. His positis,

In triangulo BAC, declinatio BC supponitur nota; angulus C est (*per 6. c. 1. h*) rectus; & angulus BAC mensuratur ab arcu NR complemento Poli altitudinis NP. Ex his tribus invenitur 1.o (*ut in 2. casu 1. schem.*) differentia ascensionalis CA, quæ (si stella Borealis sit) subtracta ascensioni rectæ CD, relinquit notam AD ascensionem obliquam. 2.o Ex iisdem (*ut in 7. casu 1. schem.*) exploratur BA amplitudo ortiva vel occidua. 3.o Addendo (si stella ad Septentrionem declinet) CA, ascensionalem differentiam Æquatoris, quadranti AÆ, habetur CÆ, eique similis BO arcus semidiurnus; adeoque & arcus diurnus; qui subtractus gradib. 360 relinquit arcum nocturnum notum.

PROBLEMA XIV.

Altitudinem Poli invenire per duas stellas simul orientes, quarum notæ sint declinationes & ascensiones rectæ.

Sit Meridianus ZKN. Æquator ÆR. Polus P. Horizon ZN, in quo sint stellarum simul orientium loca M, B. Circuli declinationum PMA. PBA. Ascensionum rectarum differentia IO, quæ notis ipsis ascensionib. nota erit.



Hinc in triangulo MPB cognoscuntur latera PM, PB, complementa datarum declinationum IM, OB.: angulus MPB mensuratur (*per 3. c. 1. h*) ab arcu noto IO. Ex his (*ut in 7. casu 2. vel 3. schem.*) invenitur angulus PBM, seu PBZ. Dein in triangulo ZPB, ex angulo Z (*per 6. c. 1. h*) recto, angulo PBZ & latere PB notis, invenitur (*ut in 1. casu 1. schem.*) latus ZP Poli P altitudo quæsita.

CANONES SINUUM ET TANGENTIUM PRO DECIMO QUOQUE MINUTO CUM EORUM LOGARITHMIS.

OPeri præstituta brevitas integros Sinuum, Tangentium, & Secantium Canones prohibebat inseri; & ex alia parte videbatur discentium exercitationi non satis prospici, si prorsùs omitterentur. Selegi proinde Sinus & Tangentes pro decimo quoque minuto, cum eorum Logarithmis; Secantibus, quarum vix est usus, penitùs prætermissis. Si quis tamen Sinus & Tangentes intermedios requirat; sic facilè investigabit.

Datis Sinubus vel Tangentibus Minutorum decimalium; invenire quoscunque intermedios.

Quæratur *v. g.* Sinus arcûs vel anguli 43. gr. 14'.

1.o Desumantur ex tabulis Sinus proximè major (43. gr. 20'. qui est 6862416) & proximè minor (43. gr. 10'. qui est 6841229) & inquiratur eorum differentia (21187.)

2.o Fiat, ut 10 ad excessum arcûs dati (43. gr. 14'.) supra arcum proximè minorem (43. gr. 10'.) id est, ut 10 ad 4: ita 21187 differentia Sinuum ex tabula desumptorum ad aliud. Quartus terminus $8474\frac{8}{10}$ seu 8475 additus Sinui proximè minori (6841229) dabit (6849704) Sinum anguli dati, vel ipsissimum qui in tabulis reperitur, vel ab illo vix differentem.

Eadem operatio institui potest ad Logarithmicos Sinus inquirendos, licèt non ita exactè, sufficienter tamen ad præsens institutum.

Quod dictum de Sinubus, de Tangentibus etiam est intelligendum. Hinc etiam & ex *Prop.* 3. *c.* 1. *P.* 1. colliges quomodò ex Sinu vel Tangente datâ inveniantur minuta correspondentia.

Gr.	Min.	Sinus.	Tangens.	Log. Sin.	Log. Tang.
0	0	0	0	0	0
	10	290.89	290.89	7.4637255	7.4637273
	20	581.77	581.78	7.7647537	7.7647610
	30	872.65	872 69	7.9408419	7.9408584
	40	1163.53	1163.61	8.0657763	8,0658057
	50	1454.39	1454.54	8.1626808	8.1627267
1	0	1745.24	1745.51	8.2418553	8.2419215
	10	2036.08	2036.50	8.3087941	8.3088842
	20	2326.90	2327.53	8.3667769	8.3668945
	30	2617 69	2618.59	8.4179190	8.4180679
	40	2908.47	2909.70	8.4636649	8.4638486
	50	3199 22	3200.86	8.5050447	8.5052671
2	0	3489.95	3492.08	8.5428192	8.5430838
	10	3780.65	3783.35	8.5775660	8.5778766
	20	4071.31	4074.69	8 6097341	8.6100943
	30	4361.94	4366.09	8.6396796	8.6400931
	40	4652.53	4657.57	8.6676893	8.6681598
	50	4943.08	4949.13	8.6939980	8.6945292
3	0	5233.60	5240.78	8 7188002	8.7193958
	10	5524.06	5532.51	8.7422586	8.7429222
	20	5814.48	5824.34	8.7645111	8.7652465
	30	6104.85	6116 26	8.7856753	8.7864861
	40	6395.17	6408.29	8.8058523	8.8067422
	50	6685.44	6700.43	8.8251299	8.8261026
4	0	6975.65	6992.68	8.8435845	8 8446437
	10	7265.80	7285.05	8.8612833	8.8624327
	20	7555.89	7577.55	6.8782854	8.8795186
	30	7845.91	7870.17	8.8946433	8.8959842
	40	8135.87	8162.93	8.9104039	8.9118460
	50	8425.76	8455.83	8.9256089	8.9271560
5	0	8715.57	8748.87	8.9402960	8.9419518
	10	9005 32	9042 06	8.9544991	8.9562672
	20	9294.99	9335.40	8.9682487	8.9701330
	30	9584 58	9628.90	8.9815729	8.9835769
	40	9874.08	9922.57	8 9944968	8.9966243
	50	10163.51	10216.41	9.0070436	9.0092984

Gr.	Min.	Sinus.	Tangens.	Log. Sin.	Log. Tang.
89	60	100000.00	Infinit.	10.0000000	Infinit.
	50	99999 58	34377371.00	9.9999982	12.5362727
	40	99998 30	17188540.00	9.9999927	12.2352390
	30	99996.19	11458865.00	9.9999835	12.0591416
	20	99993.23	8593979.10	9.9999706	11.9341943
	10	99989.42	6875008.70	9.9999541	11.8372733
88	60	99984.77	5728996.16	9.9999338	11.7580785
	50	99979.27	4910388.06	9.9999100	11.6911158
	40	99972.92	4296407.73	9.9998824	11.6331055
	30	99965.73	3818845.93	9.9998512	11.5819321
	20	99957.69	3436777.09	9.9998162	11.5361514
	10	99948.81	3124157.67	9.9997776	11.4947329
87	60	99939.08	2863625.33	9.9997354	11.4569162
	50	99928.51	2643159.96	9.9996894	11.4221234
	40	99917.09	2454175.78	9.9996398	11.3899057
	30	99904.82	2290376.55	9.9995865	11.3599059
	20	99891.71	2147040.10	9.9995297	11.3318402
	10	99877.75	2020555.35	9.9994688	11.3054708
86	60	99862.95	1908113.67	9.9994044	11.2806042
	50	99847.31	1807497.74	9.9993364	11.2570778
	40	99830.81	1716933.69	9.9992646	11.2347535
	30	99813.48	1634985.55	9.9991892	11.2135139
	20	99795.29	1560478.41	9.9991101	11.1932578
	10	99776.27	1492441.70	9.9990273	11.1738074
85	60	99756.40	1430066.63	9.9989408	11.1553563
	50	99735.69	1372673.79	9.9988506	11.1375673
	40	99714.13	1319688.30	9.9987567	11.1204714
	30	99691.73	1270620.47	9.9986591	11.1040158
	20	99668.49	1225050.55	9.9985579	11.0881540
	10	99644.40	1182616.67	9.9984529	11.0728440
84	60	99619.47	1143005.23	9.9983442	11.0580482
	50	99593.69	1105943.10	9.9982318	11.0437328
	40	99567.08	1071191.26	9.9981158	11.0298670
	30	99539.62	1038539.71	9.9979960	11.0164231
	20	99511.32	1007803.11	9.9978725	11.0033757
	10	99482.17	978817.32	9.9977453	10.9907016

Gr.	Mi.	Sinus.	Tangens.	Log. Sin.	Log. Tang.
6	0	10452.85	10510.42	9.0192346	9.0216202
	10	10742.10	10804.62	9.0310890	9.0336093
	20	11031.26	11098.99	9.0426:49	9.0452836
	30	11320.32	11393.56	9.0538588	9.0566595
	40	11609 29	11688.31	9.0648057	9.0677522
	50	11898.16	11983.28	9.0754799	9.0785760
7	0	12186.93	12278.46	9 0858945	9.0891438
	10	12475 60	12573.84	9.0960615	9.0994678
	20	12764 16	12869.43	9.1059924	9.1095894
	30	13052.62	13165.25	9.1156977	9.1194891
	40	13340.96	13461.29	9.1251872	9.1290868
	50	12629.19	13757.57	9.1344702	9.1385417
8	0	139.7.31	14054 08	9.1435553	9.1478025
	10	14205.31	14350 84	9.1524507	9.1568773
	20	14493.19	14647 84	9.1611639	9.1657737
	30	4780.94	14945 10	9.1697021	9.1744988
	40	15068 57	15242.61	9.1780721	9.1830595
	50	15356.07	15540.40	9.1862802	9.1914621
9	0	5643.45	15838 44	9.1943324	9.1997125
	10	15930.69	16136.77	9.2022345	9.2078165
	20	16217.79	16435.37	9.2099917	9.2157795
	30	16504.76	16734.26	9.2176092	9.2236065
	40	16791.59	17033 44	9.2250918	9.2313924
	50	17078.28	17332.92	9.2324440	9.2388717
10	0	17364 82	17632.70	9.2396702	9.2463188
	10	17651.21	17932 78	9.24677 6	9.2536477
	20	17937.46	18233.18	9.2537609	9.2608625
	30	18223.55	18533.90	9 2606330	9.2679669
	40	18509 44	18834.95	9 2673945	9.2749644
	50	18795.27	19136 32	9.2740487	9.2818585
11	0	19080 90	19438.03	9.2805988	9.2886523
	10	19366 36	19740.08	9.2870480	9.2953489
	20	19651 66	20042.48	9.2933993	9.3019514
	30	19936.79	20345.23	9.2996553	9.3084626
	40	20221.76	20648.34	9.3058189	9.3148851
	50	20506.55	20951.81	9.3118926	9.3212216

G.	M.	Sinus.	Tangens.	Log. Sin.	Log. Tang.
83	60	99452.18	951436.45	9 9976143	10.9783798
	50	99421 36	925530.35	9.9974797	10.9663907
	40	99389.69	900982.61	9 9973414	10.9547164
	30	99357.18	877688.74	9.9971993	10.9433405
	20	99323.83	855554.68	9.9970535	10.9322478
	10	99289.64	834495 57	9.9969040	10.9214240
82	60	99254.62	8.4434.64	9 9967507	10.9108562
	50	99218.74	795302 24	9.9965937	10.9005322
	40	99182.03	777035.06	9.9964330	10.8904406
	30	99144 49	759575.41	9.9962686	10.8805709
	20	99106.09	742870 64	9 9961004	10.8709132
	10	99066.87	726872.55	9 9959284	10.8614583
81	60	99026 80	711536.97	9 9957528	10.8521975
	50	98985.90	696823.35	9 9955734	10.8431227
	40	98944 16	682694.37	9.9953902	10.8342263
	30	98901.58	699115.62	9.9952033	10.8255012
	20	98858.17	656055 38	9 9950126	10.8169405
	10	98813.92	643484.28	9.9948181	10.8085379
80	60	98768.83	631375.15	9 9946199	10.8002875
	50	98722.91	619702.79	9 9944180	10.7921835
	40	98676.15	608443 81	9 9942122	10.7842205
	30	98628.56	597576 44	9 9940027	10.7763935
	20	98580.13	587080.42	9 9937894	10.7686976
	10	98530 87	576936.88	9 9935723	10.7611283
79	60	98480.77	567128 18	9.9933515	10.7536812
	50	98429.85	557637.86	9.9931269	10.7463523
	40	98378.08	548450 52	9.9928984	10.7391375
	30	98325.49	539551.72	9.9926661	10.7320331
	20	98272 06	530927 93	9.9924301	10.7250356
	10	98217 81	522566.47	9.9921902	10 7181415
78	60	98162 71	514455 40	9 9919466	10.7133477
	50	98106.80	506583.52	9.9916991	10 7046511
	40	98050.05	498940.27	9 9914478	10 6980486
	30	97992.47	491515 70	9.9911927	10.6915374
	20	97934.06	484300.45	9.9909338	10.6851149
	10	97874.83	477285.67	9.9906710	10.6787784

Grad.	Min.	Sinus.	Tangens.	Log. Sin.	Log. Tang.
12	0	20791.17	21255.65	9.3178789	9.3274745
	10	21075.65	21559.88	9.3237802	9.3336463
	20	21359.88	21864.48	9.3295988	9.3397391
	30	21643.96	22169.47	9.3353368	9.3457552
	40	21927.86	22474.85	9.3409963	9.3516968
	50	22211.58	22780.63	9.3465794	9.3575658
13	0	22495.11	23086.82	9.3520880	9.3633641
	10	22778.44	23393.42	9.3575240	9.3690937
	20	23061.59	23700.44	9.3628892	9.3747563
	30	23344.54	24007.87	9.3681853	9.3803537
	40	23627.29	24315.73	9.3734139	9.3858876
	50	23909.84	24624.05	9.3785762	9.3913595
14	0	24192.19	24932.80	9.3836752	9.3967711
	10	24474.33	25242.00	9.3887109	9.4021237
	20	24756.27	25551.65	9.3936852	9.4074189
	30	25038.00	25861.76	9.3985996	9.4126581
	40	25319.52	26172.34	9.4034554	9.4178425
	50	25600.82	26483.39	9.4082538	9.4229735
15	0	25881.90	26794.92	9.4129962	9.4280525
	10	26162.77	27106.93	9.4176837	9.4330804
	20	26443.42	27419.44	9.4223176	9.4380587
	30	26723.84	27732.45	9.4268988	9.4429883
	40	27004.03	28045.97	9.4314286	9.4478704
	50	27284.00	28359.99	9.4359080	9.4527061
16	0	27563.74	28674.54	9.4403381	9.4574964
	10	27843.24	28989.61	9.4447197	9.4622423
	20	28122.51	29305.21	9.4490540	9.4669448
	30	28401.53	29621.35	9.4533418	9.4716048
	40	28680.32	29938.03	9.4575840	9.4762233
	50	28958.87	30255.27	9.4617816	9.4808011
17	0	29237.17	30573.07	9.4659353	9.4853390
	10	29515.22	30891.43	9.4700461	9.4898380
	20	29793.03	31210.36	9.4741146	9.4942988
	30	30070.58	31529.88	9.4781418	9.4987223
	40	30347.88	31849.98	9.4821283	9.5031092
	50	30624.92	32170.67	9.4860749	9.5074602

		Sinus.	Tangens.	Log. Sin.	Log. Tang.
77	60	97814.76	470463.01	9.9904044	10.6725255
	50	97753.86	465824.57	9.9901339	10.6663537
	40	97692.15	461362.87	9.9898597	10.6602609
	30	97629.60	457070.85	9.9895815	10.6542448
	20	97566.23	452941.81	9.9892995	10.6483032
	10	97502.03	448969.40	9.9890137	10.6424342
76	60	97437.01	445147.59	9.9887239	10.6366358
	50	97371.16	441470.66	9.9884303	10.6309063
	40	97304.48	437933.18	9.9881329	10.6252437
	30	97236.99	434529.98	9.9878315	10.6196463
	20	97168.67	431256.14	9.9875263	10.6141124
	10	97099.54	428107.00	9.9872171	10.6086405
75	60	97029.57	425078.69	9.9869041	10.6032289
	50	96958.79	422165.18	9.9865872	10.5978763
	40	96887.18	419364.20	9.9862663	10.5925811
	30	96814.76	416671.31	9.9859416	10.5873419
	20	96741.52	414082.81	9.9856129	10.5821575
	10	96667.46	411595.19	9.9852803	10.5770265
74	60	96592.58	409205.08	9.9849438	10.5719475
	50	96516.88	406909.27	9.9846033	10.5669196
	40	96440.37	404704.67	9.9842589	10.5619413
	30	96363.05	402588.35	9.9839105	10.5570117
	20	96284.90	400557.49	9.9835582	10.5521296
	10	96205.94	398609.38	9.9832019	10.5472939
73	60	96126.17	396741.44	9.9828416	10.5425036
	50	96045.58	394951.20	9.9824774	10.5377577
	40	95964.18	393236.16	9.9821092	10.5330552
	30	95881.97	391594.34	9.9817370	10.5283952
	20	95798.95	390023.26	9.9813608	10.5237767
	10	95715.12	388520.91	9.9809805	10.5191989
72	60	95630.48	387085.26	9.9805963	10.5146610
	50	95545.02	385714.38	9.9802081	10.5101620
	40	95458.76	384406.38	9.9798158	10.5057012
	30	95371.69	383159.43	9.9794195	10.5012777
	20	95283.82	381971.94	9.9790192	10.4968908
	10	95195.14	380842.10	9.9786148	10.4925398

G.	M.	Sinus.	Tangens.	Log. Sin.	Log. Tang.
18	0	30901.70	32491.97	9.4899824	9.5117760
	10	31178.22	32813.87	9.4938513	9.5160575
	20	31454.48	33136.39	9.4976824	9.5203052
	30	31730.47	33459.53	9.5014764	9.5245199
	40	32006.19	33783.30	9.5052339	9.5287021
	50	32281.64	34107.71	9.5089556	9.5328526
19	0	32556.82	34432.76	9.5126419	9.5369718
	10	32831.72	34758.46	9.5162936	9.5410606
	20	33106.34	35084.83	9.5199112	9.5451193
	30	33380.69	35411.86	9.5234953	9.5491487
	40	33654.75	35739.56	9.5270463	9.5531492
	50	33928.53	36067.95	9.5305650	9.5571214
20	0	34202.02	36397.02	9.5340517	9.5610658
	10	34475.22	36726.80	9.5375069	9.5649831
	20	34748.13	37057.28	9.5409314	9.5688735
	30	35020.74	37388.47	9.5443253	9.5727377
	40	35293.06	37720.38	9.5476893	9.5765761
	50	35565.08	38053.03	9.5510237	9.5803892
21	0	35836.79	38386.40	9.5543292	9.5841774
	10	36108.21	38720.53	9.5576060	9.5879413
	20	36379.32	39055.41	9.5608546	9.5916812
	30	36650.13	39391.05	9.5640754	9.5953975
	40	36920.62	39727.46	9.5672689	9.5990908
	50	37190.80	40064.65	9.5704355	9.6027613
22	0	37460.66	40402.62	9.5735754	9.6064096
	10	37730.21	40741.39	9.5766892	9.6100359
	20	37999.44	41080.97	9.5797772	9.6136407
	30	38268.34	41421.36	9.5828397	9.6172243
	40	38536.93	41762.57	9.5858771	9.6207872
	50	38805.18	42104.60	9.5888897	9.6243296
23	0	39073.11	42447.49	9.5918780	9.6278519
	10	39340.71	42791.20	9.5948422	9.6313545
	20	39607.98	43135.79	9.5977827	9.6348378
	30	39874.91	43481.24	9.6006997	9.6383019
	40	40141.50	43827.56	9.6035936	9.6417473
	50	40407.75	44174.76	9.6064647	9.6451743

Gr.	Min.	Sinus.	Tangens.	Log. Sin.	Log. Tang.
71	60	95105.65	307768.35	9.9782063	10.4882240
	50	95015.36	304749.15	9.9777938	10.4839425
	40	94924.26	301783.01	9.9773772	10.4796948
	30	94832.36	298868.50	9.9769566	10.4754801
	20	94739.66	296004.22	9.9765318	10.4712979
	10	94646.16	293188.85	9.9761030	10.4671474
70	60	94551.85	290421.09	9.9756701	10.4630281
	50	94456.75	287699.70	9.9752330	10.4589394
	40	94360.85	285023.49	9.9747918	10.4548807
	30	94264.15	282391.29	9.9743466	10.4508513
	20	94166.65	279801.98	9.9738971	10.4468508
	10	94068.35	277254.48	9.9734435	10.4428786
69	60	93969.26	274747.74	9.9729858	10.4389341
	50	93869.37	272280.75	9.9725239	10.4350169
	40	93768.69	269852.54	9.9720579	10.4311265
	30	93667.22	267462.15	9.9715876	10.4272623
	20	93564.95	265108.67	9.9711132	10.4234239
	10	93461.89	262791.21	9.9706346	10.4196108
68	60	93358.04	260508.91	9.9701517	10.4158226
	50	93253.40	258260.94	9.9696647	10.4120587
	40	93147.97	256046.49	9.9691734	10.4083188
	30	93041.75	253864.78	9.9686779	10.4046025
	20	92934.75	251715 07	9.9681781	10.4009092
	10	92826.96	249596.61	9.9676741	10.3972387
67	60	92718.39	247508.69	9.9671659	10.3935904
	50	92609.03	245450.61	9.9666533	10.3899641
	40	92498.88	243421.72	9.9661365	10.3863593
	30	92387.95	241421.36	9.9656153	10.3827757
	20	92276.24	239448.89	9.9650899	10.3792128
	10	92163.75	237503.72	9.9645602	10.3756704
66	60	92050.49	235585.24	9.9640261	10.3721481
	50	91936.44	233692.87	9.9634877	10.3686455
	40	91821.61	231826.06	9.9629449	10.3651622
	30	91706.01	229984.25	9.9633978	103.616981
	20	91589.63	228166.93	9.9618463	10.3582527
	10	91472.47	226373.57	9.9612904	10.3548257

Gr.	Min.	Sinus.	Tangens.	Log. Sin.	Log. Tang.
24	0	40673.66	44522.87	9.6093133	9.6485831
	10	40939.23	44871.87	9.6121397	9.6519742
	20	41204.46	45221.79	9.6149441	9.6553477
	30	41469.32	45572.64	9.6177270	9.6587041
	40	41733.85	45924.39	9.6204884	9.6620434
	50	41998.01	46277.09	9.6232287	9.6652662
25	0	42261.83	46630.77	9.6259483	9.6686725
	10	42525.28	46985.39	9.6286472	9.6719628
	20	42788.38	47340.98	9.6313258	9.6752372
	30	43051.11	47697.55	9.6339844	9.6784961
	40	43313.4°	48055.12	9.6366231	9.6817396
	50	43575.48	48413.68	9.6392422	9.6849681
26	0	43837.12	48773.26	9.6418420	9.6881818
	10	44098.38	49133.86	9.6444226	9.6913809
	20	44359.27	49495.49	9.6469844	9.6945656
	30	44619.78	49858.16	9.6495274	9.6977363
	40	44879.92	50221.89	9.6520521	9.7008930
	50	45139.68	50586.68	9.6545584	9.7040362
27	0	45399.05	50952.54	9.6570468	9.7071659
	10	45658.04	51319.50	9.6595173	9.7102824
	20	45916.64	51687.55	9.6619701	9.7133859
	30	46174.86	52056.70	9.6644056	9.7164767
	40	46432.69	52426.98	9.6668238	9.7195549
	50	46690.12	52798.39	9.6692250	9.7226207
28	0	46947.16	53170 94	9.6716093	9.7256744
	10	47203.80	53544.65	9.6739769	9.7287161
	20	47460.04	53919.52	9.6763281	9.7317460
	30	47715.88	54295.57	9 6786629	9.7347644
	40	47971.31	54672.81	9.6809816	9.7377714
	50	48226.34	55051.25	9.6832843	9.7407672
29	0	48480.96	55430.90	9.6855712	9.7437520
	10	48735.17	55811.79	9.6878425	9.7467259
	20	48988.97	56193.91	6.6900983	9.7496892
	30	49242 36	56577.28	9.6923388	9.7526420
	40	49495.33	56961.91	9.6945642	9.7555846
	50	49747 87	57347.83	9.6967745	9.7585170

Gr.	Min.	Sinus.	Tangens.	Log. Sin.	Log. Tang.
65	60	91354.54	224603 68	9.9607302	10.3514169
	50	91235.84	222856.75	9.9601655	10.3480258
	40	91116.37	221132.34	9.9595964	10.3446523
	30	90996.13	219429.97	9.9590229	10.3412960
	20	90875.11	217749.20	9.9584450	10.3379566
	10	90753.33	216089 58	9.9578626	10.3346338
64	60	90630.78	214450.69	9.9572757	10.3313275
	50	90507.46	212832.13	9.9566844	10.3280372
	40	90383.38	211233.48	9.9560886	10.3247628
	30	90258.53	209654.36	9.9554882	10.3215039
	20	90132.91	208094 38	9.9548834	10.3182604
	10	90006.54	206553.18	9.9542741	10.3150319
63	60	89879.40	205030.38	9.9536602	10.3118182
	50	89751.51	203525.65	9.9530418	10.3086191
	40	89622.85	202038.62	9.9524188	10.3054344
	30	89493.43	200568.97	9 9517912	10.3022637
	20	89363.27	199116.37	9.9511590	10.2991070
	10	89232.33	197680.50	9.9505223	10.2959638
62	60	89100.65	196261.05	9.9498809	10.2928341
	50	88968.21	194857.71	9.9492349	10.2897176
	40	88835.02	193470.20	9.9485842	10.2866141
	30	88701.08	192098.21	9.9479289	10.2835233
	20	88566.39	190741.47	9.9472689	10.2804451
	10	88430.95	189399.71	9.9466043	10.2773793
61	60	88294.76	188072.65	9.9459349	10.2743256
	50	88157.82	186760.03	9.9452609	10.2712839
	40	88020.14	185461.59	9.9445821	10.2682540
	30	87881.71	184177.09	9.9438985	10.2652356
	20	87742.54	182906.28	9.9432102	10.2622286
	10	87602.62	181648.92	9.9425171	10.2592328
60	60	87461.97	180404.78	9.9418193	10.2562480
	50	87320.58	179173.62	9.9411166	10.2532741
	40	87178.44	177955.24	9.9404091	10.2503108
	30	87035.57	176749.40	9.9396968	10.2473580
	20	86891.96	175555 90	9.9389796	10.2444154
	10	86747.62	174374.53	9.9382576	10.2414830

Gr.	Min.	Sinus.	Tangens.	Log. Sin.	Log. Tang.
30	0	50000.00	57735.03	9.6989700	9.7614394
	10	50251.70	58123.53	9.7011508	9.7643520
	20	50502.99	58513.35	9.7033170	9.7672550
	30	50763.84	58904.50	9.7054689	9.7701485
	40	51004.26	59296.99	9.7076064	9.7730327
	50	51254.25	59690.84	9.7097299	9.7759077
31	0	51503.81	60086.06	9.7118393	9.7787737
	10	51752.93	60482.66	9.7139349	9.7816309
	20	52001.61	60880.67	9.7160168	9.7844794
	30	52249.86	61280.08	9.7180851	9.7873193
	40	52497.66	61680.92	9.7201399	9.7901508
	50	52745.02	62083.20	9.7221814	9.7929741
32	0	52991.93	62486.94	9.7242097	9.7957892
	10	53238.39	62892.15	9.7262249	9.7985964
	20	53484.40	63298.83	9.7282271	9.8013957
	30	53729.96	63707.03	9.7302165	9.8041873
	40	53975.07	64116.73	9.7321932	9.8069714
	50	54219.71	64527.97	9.7341572	9.8097480
33	0	54463.90	64940.76	9.7361088	9.8125174
	10	54707.63	65355.11	9.7380479	9.8152795
	20	54950.90	65771.03	9.7399748	9.8180347
	30	55193.70	66188.56	9.7418895	9.8207829
	40	55436.03	66607.69	9.7437921	9.8235244
	50	55677.90	67028.45	9.7456828	9.8262592
34	0	55919.29	67450.85	9.7475617	9.8289874
	10	56160.21	67874.92	9.7494287	9.8317093
	20	56400.65	68300.66	9.7512842	9.8344249
	30	56640.62	68728.10	9.7531280	9.8371343
	40	56880.11	69157.24	9.7549604	9.8398377
	50	57119.12	69588.13	9.7567815	9.8425351
35	0	57357.64	70020.75	9.7585913	9.8452268
	10	57595.68	70455.15	9.7603899	9.8479127
	20	57833.23	70891.33	9.7621775	9.8505931
	30	58070.30	71329.31	9.7639540	9.8532680
	40	58306.87	71769.11	9.7657197	9.8559376
	50	58542.94	72210.75	9.7674746	9.8586019

Q.	M.	Sinus.	Tangens.	Log. Sin.	Log. Tang.
59	60	86602.54	173205.08	9.9375306	10.2385606
	50	86456.73	172047.36	9.9367988	10.2356480
	40	86310.19	170901.16	9.9360621	10.2327450
	30	86162.92	169766.31	9.9353204	10.2298515
	20	86014.91	168642.61	9.9345738	10.2269673
	10	85866.18	167529.88	9.9338222	10.2240923
58	60	85716.73	166427.95	9.9330656	10.2212263
	50	85566.55	165336.63	9.9323040	10.2183691
	40	85415.64	164255.76	9.9315374	10.2155206
	30	85264.02	163185.17	9.9307658	10.2126807
	20	85111.66	162124.69	9.9299891	10.2098492
	10	84958.60	161074.17	9.9292073	10.2070259
57	60	84804.81	160033.45	9.9284205	10.2042108
	50	84650.30	159002.38	9.9276285	10.2014036
	40	84495.08	157980.79	9.9268314	10.1986043
	30	84339.12	156968.56	9.9260292	10.1958127
	20	84182.49	155965.52	9.9252218	10.1930286
	10	84025.13	154971.55	9.9244092	10.1902520
56	60	83867.06	153986.50	9.9235914	10.1874826
	50	83708.27	153010.23	9.9227684	10.1847205
	40	83548.78	152042.61	9.9219401	10.1819653
	30	83388.58	151083.52	9.9211066	10.1792171
	20	83227.68	150132.82	9.9202678	10.1764756
	10	83066.07	149190.38	9.9194237	10.1737408
55	60	82903.76	148256.10	9.9185742	10.1710126
	50	82740.74	147329.83	9.9177194	10.1682907
	40	82577.03	146411.47	9.9168593	10.1655751
	30	82412.62	145500.90	9.9159937	10.1628657
	20	82247.51	144598.01	9.9151228	10.1601623
	10	82081.70	143702.68	9.9142464	10.1574649
54	60	81915.21	142814.80	9.9133645	10.1547732
	50	81748.01	141934.27	9.9124772	10.1520873
	40	81580.13	141060.98	9.9115844	10.1494069
	30	81411.55	140194.83	9.9106860	10.1467320
	20	81242.29	139335.71	9.9097821	10.1440624
	10	81072.33	138483.53	9.9088727	10.1413981

Gr.	Min.	Sinus.	Tangens.	Log. Sin.	Log. Tang.
36	0	58778.53	72654.26	9.7692187	9.8612610
	10	59013.61	73099.63	9.7709522	9.8639152
	20	59248.19	73546.91	9.7726751	9.8665644
	30	59482.28	73996.11	9.7743876	9.8692089
	40	59715.86	74447.24	9.7760897	9.8718486
	50	59948.93	74900.33	9.7777815	9.8744838
37	0	60181.50	75355.41	9.7794630	9.8771144
	10	60413.56	75812.48	9.7811344	9.8797407
	20	60645.11	76271.57	9.7827958	9.8823627
	30	60876.14	76732.70	9.7844471	9.8849805
	40	61106.66	77195.89	9.7860886	9.8875942
	50	61336.66	77661.17	9.7877202	9.8902040
38	0	61566.15	78128.56	9.7893420	9.8928098
	10	61795.11	78598.08	9.7909541	9.8954119
	20	62023.55	79069.75	9.7925566	9.8980104
	30	62251.46	79543.59	9.7941496	9.9006052
	40	62478.85	80019.63	9.7957330	9.9031966
	50	62705.71	80497.90	9.7973071	9.9057845
39	0	62932.04	80978.40	9.7988718	9.9083692
	10	63157.84	81461.18	9.8004272	9.9109507
	20	63383.09	81946.25	9.8019735	9.9135291
	30	63607.82	82433.64	9.8035105	9.9161045
	40	63842.01	82923.37	9.8050385	9.9186769
	50	64055.66	83415.47	9.8065575	9.9212466
40	0	64278.76	83909.96	9.8080675	9.9238135
	10	64501.32	84406.88	9.8095686	9.9263778
	20	64723.34	84906.24	9.8110609	9.9289396
	30	64944.80	85408.07	9.8125444	9.9314989
	40	65165.72	85912.40	9.8140192	9.9340559
	50	65386.09	86419.26	9.8154854	9.9366105
41	0	65605.90	86928.68	9.8169429	9.9391631
	10	65825.16	87440.67	9.8183919	9.9417135
	20	66043.86	87955.28	9.8198325	9.9442619
	30	66262.01	88472.53	9.8212646	9.9468084
	40	66479.59	88992.45	9.8226883	9.9493531
	50	66696.61	89515.06	9.8241037	9.9518961

Gr.	Min.	Sinus.	Tangens.	Log. Sin.	Log. Tang.
53	60	80901.70	137638.19	9.9079576	10.1387390
	50	80730.38	136799.59	9.9070370	10.1360848
	40	80558.37	135967.64	9.9061107	10.1334356
	30	80385.69	135142.24	9.9051787	10.1307911
	20	80212.32	134323.31	9.9042411	10.1281514
	10	80038.27	133510.75	9.9032977	10.1255162
52	60	79863.55	132704.48	9.9023486	10.1228856
	50	79688.15	131904.41	9.9013938	10.1202593
	40	79512.08	131110.46	9.9004331	10.1176373
	30	79335.33	130322.54	9.8994667	10.1150195
	20	79157.92	129540.57	9.8984944	10.1124058
	10	78979.83	128764.47	9.8975162	10.1097960
51	60	78801.07	127994.16	9.8965321	10.1071902
	50	78621.65	127229.57	9.8955422	10.1045881
	40	78441.57	126470.62	9.8945463	10.1019896
	30	78260.82	125717.23	9.8935444	10.0993948
	20	78079.40	124969.33	9.8925365	10.0968034
	10	77897.33	124226.85	9.8915226	10.0942155
50	60	77714.60	123489.72	9.8905026	10.0916308
	50	77531.21	122757.86	9.8894765	10.0890493
	40	77347.16	122031.21	9.8884444	10.0864709
	30	77162.46	121309.70	9.8874061	10.0838955
	20	76977.10	120593.27	9.8863616	10.0813231
	10	76791.10	119881.84	9.8853109	10.0787534
49	60	76604.44	119175.36	9.8842540	10.0761865
	50	76417.14	118473.76	9.8831908	10.0736222
	40	76229.19	117776.98	9.8821213	10.0710604
	30	76040.60	117084.96	9.8810455	10.0685011
	20	75851.36	116397.63	9.8799634	10.0659441
	10	75661.47	115714.95	9.8788748	10.0633895
48	60	75470.97	115036.84	9.8777799	10.0608369
	50	75279.80	114363.26	9.8766785	10.0582865
	40	75088.00	113694.14	9.8755706	10.0557381
	30	74895.57	113029.44	9.8744561	10.0531916
	20	74702.51	112369.09	9.8733352	10.0506469
	10	74508.81	111713.05	9.8722076	10.0481039

°	′	Sinus.	Tangens.	Log. Sin.	Log. Tang.
42	0	66913.96	90040.61	9.8255109	9.9144374
	10	67128.95	90568.61	9.8269998	9.9159772
	20	67344.27	91099.41	9.8283906	9.9175155
	30	67559.02	91633.12	9.8296833	9.9190525
	40	67773.20	92169.68	9.8310580	9.9205881
	50	67986.81	92709.14	9.8324246	9.9221225
43	0	68199.84	93251.51	9.8337833	9.9236559
	10	68412.29	93796.83	9.8351341	9.9251882
	20	68624.16	94345.13	9.8364771	9.9267195
	30	68835.45	94896.46	9.8378122	9.9282500
	40	69046.17	95450 83	9.8391396	9.9297797
	50	69256.30	96008.29	9.8404593	9.9313087
44	0	69465.84	96568.88	9.8417713	9.9328372
	10	69674.79	97132.62	9.8430757	9.9343651
	20	69883.15	97699.56	9.8443725	9.9358926
	30	70090.93	98269.73	9.8456618	9.9374197
	40	70298.10	98843.16	9.8469436	9.9389466
	50	70504.69	99419.91	9.8482180	9.9374734
45	0	70710.68	100000.00	9.8494850	10.0000000

In Tabulis his Secantes prætermisimus, tum quia in ea, quam secuti sumus, methodo nunquam adhibentur: tum quia notis complementorum sinubus nullo negotio possunt investigari.

Dato Sinu complementi cujusvis arcus vel anguli, invenire illius Secantem.

Divide quadratum Sinûs totius per Sinum complementi; quotiens erit Secans quæsita. *Ratio patet ex p. 2. c. 1. p. 1.*

Exemplum. Quæratur Secans anguli 60 gr. Quadratum radii, seu 1000000000000000, divide per 5000000, sinum 30 gr. seu complementi. Quotiens 200000000 erit Secans quæsita.

Si per Logarithmos opereris, ex Logarithmo radii duplicato, subtrahe Logarithmum Sinûs complementi. Residuum erit Secans quæsita.

Idem obtinebis dicendo: Ut sinus anguli dati, ad sinum totum: sic tangens anguli dati, ad secantem quæsitam.

°	′	Sinus.	Tangens.	Log. Sin.	Log. Tang.
47	60	74314.48	111061.25	9.8710735	10.0455626
	50	74119.53	110413.65	9.8699326	10.0430228
	40	73923.94	109770.20	9.8687851	10.0404845
	30	73727.73	109130.85	9.8676309	10.0379475
	20	73530.90	108495.54	9.8664699	10.0354119
	10	73333.45	107864.23	9.8653021	10.0328775
46	60	73135.37	107236.87	9.8641275	10.0303441
	50	72936.67	106613.41	9.8629460	10.0278118
	40	72737.36	105993.81	9.8617576	10.0252805
	30	72537.44	105378.01	9.8605622	10.0227500
	20	72336.90	104765.98	9.8593599	10.0202203
	10	72135.74	104157.67	9.8581505	10.0176913
45	60	71933.98	103553.03	9.8569341	10.0151628
	50	71731.61	102952.03	9.8557106	10.0126349
	40	71528.63	102354.61	9.8544799	10.0101074
	30	71325.05	101760.74	9.8532421	10.0075803
	20	71120.86	101170.37	9.8519970	10.0050538
	10	70916.07	100583.47	9.8507446	10.0025266
44	60	70710.68	100000.00	9.8494850	10.0000000

Moneri hîc etiam debet Lector, quòd tametsi in Tabulis computari soleant Sinus, Tangentes & Secantes pro Sinu toto 10000000: abjectis tamen duabus notis prioribus, habeantur Sinus &c. ad radium 100000, quales ad operationes obviæ satis accuratè peragendas omninò sufficiunt. Quænam verò notæ sint abjiciendæ, indicat punctum quod postponi solet duabus primis notis à dextra vel expressis vel intellectis.

Dixi *expressis vel intellectis*; quia si punctum, ut quandoque fit, sequatur immediatè primam notam, indicium est deesse unam: si verò præfigatur primæ, ostendit desiderari duas notas, quibus totidem Cyphræ sunt substituendæ. *Ex. gr.* Si in Tabulis deprehendatur Tangens 8814357. 2, adjici debet una Cyphra, sic 88143570: vel si inveniatur Secans 85943689. præfigendæ sunt duæ Cyphræ, sic 8594368900, ut habeatur Tangens vel Secans integra pro radio 10000000.

TABULA LOGARITHMORUM,

Pro Numeris naturali serie crescentibus ab Unitate ad 1000.

QUæ ad Logarithmorum naturam, inventionem, usumque pertinent, videre poterit studiosus Lector in Arithmetica methodicè succinctèque tradita Cap. 7. ubi (§. 3. & 4.) demonstratur modus facilis inveniendi cujuscunque numeri, sive fracti, sive integri Logarithmum; & (§. 5.) numerum cuicunque Logarithmo dato correspondentem. Nec refert quòd Regulæ ibidem traditæ adaptentur præcipuè Tabulis majoribus continentibus Logarithmos numerorum naturali serie crescentium ab unitate ad 10000, quales passim extant; nam quicunque Regulas illas, & illarum demonstrationes intellexerit, quæ istic præcipiuntur, sequenti Tabulæ facilè applicabit.

Aliud etiam est, quod si à Lectore observetur, Tabulæ hujus Logarithmicæ brevitatem aliquatenùs commensabit; si nimirum quando numerus proponitur excedens maximum in Tabula, eum (*per Reductionis Regulam*) convertat in alium minorem, sed æquivalentem in specie diversa: *Ex. gr.* Si in triangulo aliquo resolvendo detur latus continens pedes 2400, reducantur pedes ad ulnas 800, vel passus 480, vel hexapedas 400, vel decempedas 240. Et per numeri sic reducti Logarithmum perficiatur operatio.

Quòd si contingat ut post reductionem superstit aliqua fractio, quomodò inveniatur Logarithmus huic correspondens in Arithmetica indicatur.

N.	Logarith.
1	0.0000000
2	0.3010300
3	0.4771212
4	0.6020600
5	0.6989700
6	0.7781512
7	0.8450980
8	0.9030900
9	0.9542425
10	1.0000000
11	1.0413927
12	1.0791812
13	1.1139433
14	1.1461280
15	1.1760913
16	1.2041200
17	1.2304489
18	1.2552725
19	1.2787531
20	1.3010300
21	1.3222193
22	1.3424227
23	1.3617278
24	1.3802112
25	1.3979400
26	1.4149733
27	1.4313638
28	1.4471580
29	1.4623980
30	1.4771212
31	1.4913617
32	1.5051500
33	1.5185139
34	1.5314789
35	1.5440680
36	1.5563025
37	1.5682017
38	1.5797836
39	1.5910646
40	1.6020600
41	1.6127839
42	1.6232493
43	1.6334685
44	1.6434527
45	1.6532125
46	1.6627578
47	1.6720979
48	1.6812412
49	1.6901961
50	1.6989700
51	1.7075702
52	1.7160033
53	1.7242759
54	1.7323938
55	1.7403627
56	1.7481880
57	1.7558748
58	1.7634280
59	1.7708520
60	1.7781512
61	1.7853298
62	1.7923917
63	1.7993405
64	1.8061800
65	1.8129133
66	1.8195439
67	1.8260748
67	1.8260748
68	1.8325089
69	1.8388491
70	1.8450980
71	1.8512583
72	1.8573325
73	1.8633229
74	1.8692317
75	1.8750613
76	1.8808136
77	1.8864907
78	1.8920946
79	1.8976271
80	1.9030900
81	1.9084850
82	1.9138138
83	1.9190781
84	1.9242793
85	1.9294189
86	1.9344984
87	1.9395192
88	1.9444827
89	1.9493900
90	1.9542425
91	1.9590414
92	1.9637878
93	1.9684829
94	1.9731278
95	1.9777236
96	1.9822712
97	1.9867717
98	1.9912761
99	1.9956352
100	2.0000000

100

N.	Logarith.	N.	Logarith.	N.	Logarith.
101	2.0043214	134	2.1271048	167	2.2227165
102	2.0086002	135	2.1303338	168	2.2253093
103	2.0128372	136	2.1335389	169	2.2278867
104	2.0170333	137	2.1367206	170	2.2304489
105	2.0211893	138	2.1398791	171	2.2329961
106	2.0253059	139	2.1430148	172	2.2355284
107	2.0293838	140	2.1461280	173	2.2380461
108	2.0334238	141	2.1492191	174	2.2405492
109	2.0374265	142	2.1522883	175	2.2430380
110	2.0413927	143	2.1553360	176	2.2455127
111	2.0453230	144	2.1583625	177	2.2479733
112	2.0492180	145	2.1613680	178	2.2504200
113	2.0530784	146	2.1643528	179	2.2528530
114	2.0569048	147	2.1673173	180	2.2552725
115	2.0606978	148	2.1702617	181	2.2576786
116	2.0644580	149	2.1731863	182	2.2600714
117	2.0681859	150	2.1760913	183	2.2624511
118	2.0718820	151	2.1789769	184	2.2648178
119	2.0755470	152	2.1818436	185	2.2671717
120	2.0791812	153	2.1846914	186	2.2695129
121	2.0827854	154	2.1875207	187	2.2718418
122	2.0863598	155	2.1903317	188	2.2741578
123	2.0899051	156	2.1931246	189	2.2764618
124	2.0934217	157	2.1958996	190	2.2787536
125	2.0969100	158	2.1986571	191	2.2810334
126	2.1003705	159	2.2013971	192	2.2833012
127	2.1038037	160	2.2041200	193	2.2855573
128	2.1072100	161	2.2068259	194	2.2878017
129	2.1105897	162	2.2095150	195	2.2900346
130	2.1139433	163	2.2121876	196	2.2922561
131	2.1172713	164	2.2148438	197	2.2944662
132	2.1205739	165	2.2174839	198	2.2966652
133	2.1238516	166	2.2201081	199	2.2988531
134	2.1271048	167	2.2227165	200	2.3010300

200

N.	Logarith.	N.	Logarith.	N.	Logarith.
201	2.3031961	234	2.3692159	267	2.4265113
202	2.3053514	235	2.3710679	268	2.4281348
203	2.3074960	236	2.3729120	269	2.4297523
204	2.3096302	237	2.3747483	270	2.4313638
205	2.3117539	238	2.3765770	271	2.4329693
206	2.3138672	239	2.3783979	272	2.4345689
207	2.3159703	240	2.3802112	273	2.4361626
208	2.3180633	241	2.3820170	274	2.4377506
209	2.3201463	242	2.3838154	275	2.4393327
210	2.3222193	243	2.3856063	276	2.4409091
211	2.3242824	244	2.2873898	277	2.4424798
212	2.3263359	245	2.3891661	278	2.4440448
213	2.3283796	246	2.3909351	279	2.4456042
214	2.3304138	247	2.3926969	280	2.4471580
215	2.3324385	248	2.3944517	281	2.4487063
216	2.3344537	249	2.3961993	282	2.4502491
217	2.3364597	250	2.3979400	283	2.4517864
218	2.3384565	251	2.3996737	284	2.4533183
219	2.3404441	252	2.4014005	285	2.4548449
220	2.3424227	253	2.4031205	286	2.4563660
221	2.3443923	254	2.4048337	287	2.4578819
222	2.3463530	255	2.4065402	288	2.4593925
223	2.3483049	256	2.4082400	289	2.4608978
224	2 3502480	257	2.4099331	290	2.4623980
225	2.3521825	258	2.4116197	291	2.4638930
226	2.3541084	259	2.4132998	292	2.4653828
227	2.3560259	260	2.4149733	293	2.4668676
228	2.3579348	261	2.4166405	294	2.4683473
229	2.3598355	262	2.4183013	295	2.4698220
230	2.3617278	263	2.4199557	296	2.4712917
231	2.3636120	264	2.4216039	297	2.4727564
232	2.3654880	265	2.4232459	298	2.4742163
233	2.3673559	266	2.4248816	299	2.4756712
234	2.3692159	267	2.4265113	300	2.4771212

300

N.	Logarith.	N.	Logarith.	N.	Logarith.
301	2.4785665	334	2.5237465	367	2.5646661
302	2.4800069	335	2.5250448	368	2.5658478
303	2.4814426	336	2.5263393	369	2.5670264
304	2.4828736	337	2.5276299	370	2.5682017
405	2.4842998	338	2.5289167	371	2.5693739
306	2.4857214	339	2.5301997	372	2.5705429
307	2.4871384	340	2.5314789	373	2.5717088
308	2.4885507	341	2.5327544	374	2.5728716
309	2.4899585	342	2.5340261	375	2.5740313
310	2.4913617	343	2.5352941	376	2.5751878
311	2.4927604	344	2.5365584	377	2.5763413
312	2.4941546	345	2.5378191	378	2.5774918
313	2.4955443	346	2.5390761	379	2.5786392
314	2.4969296	347	2.5403295	380	2.5797836
315	2.4983105	348	2.5415792	381	2.5809250
316	2.4996871	349	2.5428254	382	2.5820634
317	2.5010593	350	2.5440680	383	2.5831988
318	2.5024271	351	2.5453071	384	2.5843312
319	2.5037907	352	2.5465427	385	2.5854607
320	2.5051500	353	2.5477747	386	2.5865873
321	2.5065050	354	2.5490033	387	2.5877110
322	2.5078559	355	2.5502283	388	2.5888317
323	2.5092025	356	2.5514500	389	2.5899496
324	2.5105450	357	2.5526682	390	2.5910646
325	2.5118834	358	2.5538830	391	2.5921768
326	2.5132176	359	2.5550944	392	2.5932861
327	2.5145477	360	2.5563025	393	2.5943925
328	2.5158738	361	2.5575072	394	2.5954962
329	2.5171959	362	2.5587086	395	2.5965971
330	2.5185139	363	2.5599066	396	2.5976952
331	2.5198280	364	2.5611014	397	2.5987905
332	2.5211381	365	2.5622929	398	2.5998831
333	2.5224442	366	2.5634811	399	2.6009729
334	2.5237465	367	2.5646661	400	2.6020600

400

N.	Logarith.
401	2.6031444
402	2.6042260
403	2.6053050
404	2.6063814
405	2.6074550
406	2.6085260
407	2.6095944
408	2.6106602
409	2.6117233
410	2.6127839
411	2.6138418
412	2.6148972
413	2.6159500
414	2.6170003
415	2.6180481
416	2.6190933
417	2.6201360
418	2.6211763
419	2.6222140
420	2.6232493
421	2.6242821
422	2.6253124
423	2.6263404
424	2.6273659
425	2.6283889
426	2.6294096
427	2.6304279
428	2.6314438
429	2.6324573
430	2.6334685
431	2.6344773
432	2.6354837
433	2.6364879
434	2.6374897

N.	Logarith.
434	2.6374897
435	2.6384893
436	2.6394865
437	2.6404814
438	2.6414741
439	2.6424645
440	2.6434527
441	2.6444386
442	2.6454223
443	2.6464037
444	2.6473830
445	2.6483600
446	2.6493349
447	2.6503075
448	2.6512780
449	2.6522463
450	2.6532125
451	2.6541765
452	2.6551384
453	2.6560982
454	2.6570558
455	2.6580114
456	2.6589648
457	2.6599162
458	2.6608655
459	2.6618127
460	2.6627578
461	2.6637009
462	2.6646420
463	2.6655810
464	2.6665180
465	2.6674529
466	2.6683859
467	2.6693169

N.	Logarith.
467	2.6693169
468	2.6702458
469	2.6711728
470	2.6720979
471	2.6730209
472	2.6739420
473	2.6748611
474	2.6757783
475	2.6766936
476	2.6776069
477	2.6785184
478	2.6794279
479	2.6803355
480	2.6812412
481	2.6821451
482	2.6830470
483	2.6839471
484	2.6848454
485	2.6857417
486	2.6866363
487	2.6875290
488	2.6884198
489	2.6893089
490	2.6901961
491	2.6910815
492	2.6919651
493	2.6928469
494	2.6937269
495	2.6946052
496	2.6954817
497	2.6963564
498	2.6972293
499	2.6981005
500	2.6989700

500

N.	Logarith.	N.	Logarith.	N.	Logarith.
501	2.6998377	534	2.7275413	567	2.7535831
502	2.7007037	535	2.7283538	568	2.7543483
503	2.7015680	536	2.7291648	569	2.7551123
504	2.7024305	537	2.7299743	570	2.7558748
505	2.7032914	538	2.7307823	571	2.7566361
506	2.7041503	539	2.7315888	572	2.7573960
507	2.7050080	540	2.7323938	573	2.7581546
508	2.7058637	541	2.7331973	574	2.7589119
509	2.7067178	542	2.7339993	575	2.7596678
510	2.7075702	543	2.7347998	576	2.7604225
511	2.7084209	544	2.7355989	577	2.7611758
512	2.7092700	545	2.7363965	578	2.7619278
513	2.7101174	546	2.7371926	579	2.7626786
514	2.7109631	547	2.7379873	580	2.7634280
515	2.7118072	548	2.7387806	581	2.7641761
516	2.7126497	549	2.7395723	582	2.7649230
517	2.7134905	550	2.7403627	583	2.7656685
518	2.7143298	551	2.7411516	584	2.7664128
519	2.7151674	552	2.7419391	585	2.7671559
520	2.7160033	553	2.7427251	586	2.7678976
521	2.7168377	554	2.7435098	587	2.7686381
522	2.7176705	555	2.7442930	588	2.7693773
523	2.7185017	556	2.7450748	589	2.7701153
524	2.7193313	557	2.7458552	590	2.7708520
525	2.7201593	558	2.7466342	591	2.7715875
526	2.7209857	559	2.7474118	592	2.7723217
527	2.7218106	560	2.7481880	593	2.7730547
528	2.7226339	561	2.7489629	594	2.7737864
529	2.7234557	562	2.7497363	595	2.7745170
530	2.7242759	563	2.7505084	596	2.7752463
531	2.7250945	564	2.7512791	597	2.7759743
532	2.7259116	565	2.1520484	598	2.7767012
533	2.7267272	566	2.7528164	599	2.7774268
534	2.7275413	567	2.7535831	600	2.7781512

N.	Logarith.	N.	Logarith.	N.	Logarith.
601	2.7788745	634	2.8020093	667	2.8241258
602	2.7795965	635	2.8027737	668	2.8247765
603	2.7803173	636	2.8034571	669	2.8254261
604	2.7810369	637	2.8041394	670	2.8260748
605	2.7817554	638	2.8048207	671	2.8267225
606	2.7824726	639	2.8055009	672	2.8273693
607	2.7831887	640	2.8061800	673	2.8280151
608	2.7839036	641	2.8068580	674	2.8286599
609	2.7846173	642	2.8075350	675	2.8293038
610	2.7853298	643	2.8082110	676	2.8299467
611	2.7860412	644	2.8088859	677	2.8305887
612	2.7867514	645	2.8095597	678	2.8312297
613	2.7874605	646	2.8102325	679	2.8318698
614	2.7881684	647	2.8109043	680	2.8325089
615	2.7888751	648	2.8115750	681	2.8331471
616	2.7895807	649	2.8122447	682	2.8337844
617	2.7902852	650	2.8129134	683	2.8344207
618	2.7909885	651	2.8135810	684	2.8350561
619	2.7916906	652	2.8142476	685	2.8356906
620	2.7923917	653	2.8149132	686	2.8363241
621	2.7930916	654	2.8155777	687	2.8369567
622	2.7937904	655	2.8162413	688	2.8375884
623	2.7944880	656	2.8169038	689	2.8382192
624	2.7951846	657	2.8175654	690	2.8388491
625	2.7958800	658	2.8182259	691	2.8394780
626	2.7965744	659	2.8188854	692	2.8401061
627	2.7972675	660	2.8195439	693	2.8407332
628	2.7979596	661	2.8202015	694	2.8413595
629	2.7986506	662	2.8208580	695	2.8419848
630	2.7993405	663	2.8215135	696	2.8426092
631	2.8000294	664	2.8221681	697	2.8432328
632	2.8007171	665	2.8228216	698	2.8438554
633	2.8014037	666	2.8234742	699	2.8444772
634	2.8020893	667	2.8241258	700	2.8450980

N	Logarith.
700	
701	2.8457180
702	2.8463371
703	2.8469553
704	2.8475727
705	2.8481891
706	2.8488047
707	2.8494194
708	2.8500333
709	2.8506462
710	2.8512583
711	2.8518696
712	2.8524800
713	2.8530895
714	2.8536982
715	2.8543060
716	2.8549130
717	2.8555191
718	2.8561244
719	2.8567289
720	2.8573325
721	2.8579353
722	2.8585372
723	2.8591383
724	2.8597386
725	2.8603380
726	2.8609366
727	2.8615344
728	2.8621314
729	2.8627275
730	2.8633229
731	2.8639174
732	2.8645111
733	2.8651040
734	2.8656961

N.	Logarith.
734	2.8656961
735	2.8662873
736	2.8668778
737	2.8674675
738	2.8680564
739	2.8686444
740	2.8692317
741	2.8698182
742	2.8704039
743	2.8709888
744	2.8715729
745	2.8721563
746	2.8727388
747	2.8733206
748	2.8739016
749	2.8744818
750	2.8750613
751	2.8756399
752	2.8762178
753	2.8767950
754	2.8773713
755	2.8779469
756	2.8785218
757	2.8790959
758	2.8796692
759	2.8802418
760	2.8808136
761	2.8813847
762	2.8819550
763	2.8825245
764	2.8830934
765	2.8836614
766	2.8842288
767	2.8847954

N.	Logarith.
767	2.8847954
768	2.8853612
769	2.8859263
770	2.8864907
771	2.8870544
772	2.8876173
773	2.8881795
774	2.8887410
775	2.8893017
776	2.8898617
777	2.8904210
778	2.8909796
779	2.8915375
780	2.8920946
781	2.8926510
782	2.8932067
783	2.8937618
784	2.8943161
785	2.8948698
786	2.8954225
787	2.8959747
788	2.8965261
789	2.8970770
790	2.8976271
791	2.8981765
792	2.8987252
793	2.8992732
794	2.8998205
795	2.9003671
796	2.9009131
797	2.9014583
798	2.9020029
799	2.9025468
800	2.9030900

800

N.	Logarith.
801	2.9036325
802	2.9041744
803	2.9047155
804	2.9052560
805	2.9057959
806	2.9063350
807	2.9068735
808	2.9074114
809	2.9079485
810	2.9084850
811	2.9090208
812	2.9095560
813	2.9100905
814	2.9106244
815	2.9111576
816	2.9116901
817	2.9122220
818	2.9127533
819	2.9132839
820	2.9138138
821	2.9143431
822	2.9148718
823	2.9153998
824	2.9159272
825	2.9164539
826	2.9169800
827	2.9175055
828	2.9180303
829	2.9185545
830	2.9190781
831	2.9196010
832	2.9201233
833	2.9206450
834	2.9211660

N.	Logarith.
834	2.9211660
835	2.9216865
836	2.9222063
837	2.9227254
838	2.9232440
839	2.9237620
840	2.9242793
841	2.9247960
842	2.9253121
843	2.9258276
844	2.9263424
845	2.9268567
846	2.9273704
847	2.9278834
848	2.9283958
849	2.9289077
850	2.9294189
851	2.9299296
852	2.9304396
853	2.9309490
854	2.9314579
855	2.9319661
856	2.9324738
857	2.9329808
858	2.9334873
859	2.9339932
860	2.9344984
861	2.9350031
862	2.9355073
863	2.9360108
864	2.9365137
865	2.9370161
866	2.9375179
867	2.9380191

N.	Logarith.
867	2.9380191
868	2.9385197
869	2.9390198
870	2.9395192
871	2.9400181
872	2.9405165
873	2.9410142
874	2.9415114
875	2.9420080
876	2.9425041
877	2.9429996
878	2.9434945
879	2.9439889
880	2.9444827
881	2.9449759
882	2.9454686
883	2.9459607
884	2.9464523
885	2.9469433
886	2.9474337
887	2.9479236
888	2.9484130
889	2.9489018
890	2.9493900
891	2.9498777
892	2.9503648
893	2.9508514
894	2.9513375
895	2.9518230
896	2.9523080
897	2.9527924
898	2.9532763
899	2.9537597
900	2.9542425

900

N.	Logarith.
901	2.9547248
902	2.9552065
903	2.9556877
904	2.9561684
905	2.9566486
906	2.9571282
907	2.9576073
908	2.9580858
909	2.9585639
910	2.9590414
911	2.9595184
912	2.9599948
913	2.9604708
914	2.9609462
915	2.9614211
916	2.9618955
917	2.9623693
918	2.9628427
919	2.9633155
920	2.9637878
921	2.9642596
922	2.9647309
923	2.9652017
924	2.9656720
925	2.9661417
926	2.9666110
927	2.9670797
928	2.9675480
929	2.9680157
930	2.9684829
931	2.9689497
932	2.9694159
933	2.9698816
934	2.9703469

N.	Logarith.
934	2.9703469
935	2.9708116
936	2.9712758
937	2.9717396
938	2.9722028
939	2.9726656
940	2.9731278
941	2.9735896
942	2.9740509
943	2.9745117
944	2.9749720
945	2.9754318
946	2.9758911
947	2.9763500
948	2.9768083
949	2.9772662
950	2.9777236
951	2.9781805
952	2.9786369
953	2.9790929
954	2.9795484
955	2.9800034
956	2.9804579
957	2.9809119
958	2.9813655
959	2.9818186
960	2.9822712
961	2.9827234
962	2.9831751
963	2.9836263
964	2.9840770
965	2.9845273
966	2.9849771
967	2.9854265

N.	Logarith.
967	2.9854265
968	2.9858753
969	2.9863238
970	2.9867717
971	2.9872192
972	2.9876663
973	2.9881128
974	2.9885589
975	2.9890046
976	2.9894498
977	2.9898946
978	2.9903388
979	2.9907827
980	2.9912261
981	2.9916690
982	2.9921115
983	2.9925535
984	2.9929951
985	2.9934362
986	2.9938769
987	2.9943171
988	2.9947569
989	2.9951963
990	2.9956352
991	2.9960736
992	2.9965117
993	2.9969492
994	2.9973864
995	2.9978231
996	2.9982593
997	2.9986951
998	2.9991305
999	2.9995655
1000	3.0000000

PRÆCIPUA GEOMETRIÆ PLANÆ ELEMENTA

EX EUCLIDE SELECTA

ET

Incipientium usui accommodata.

Tametsi in Propositionum delectu praecipua cura fuit, ne qua deesset Trigonometriæ demonstrandæ: simul tamen dedi operam, ut Opus hoc ad alia quàm plurima inserviret; ideoque adjunxi ea, quæ videbantur ad reliquam Mathesim maximè usui futura. Quoad Propositionum ordinem, eum tenui, qui ab Euclide ejusque Interpretibus observatus. Corollaria verò adjeci illa quæ videbantur incipientibus magis necessaria.

EX LIBRO PRIMO.

DEFINITIONES.

1. *Punctum* est magnitudinis initium nullam habens dimensionem.

2. *Linea* est magnitudo solam habens longitudinem.

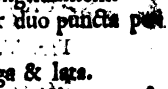
4. *Linea Recta* est brevissima quæ inter duo puncta potest duci.

5. *Superficies* est magnitudo tantùm longa & lata.

7. *Superficies Plana* est minima omnium eosdem terminos habentium.

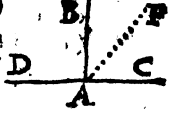
Corpus est magnitudo longa, lata, & profunda.

8. *Angulus* est duarum linearum (BE, BF) se mutuò tangentium, alterius ad alterum inclinatio, seu (ut alii malunt) alterius ab altero divaricatio: dicitur *Rectilineus*, *Curvilineus*, vel *Mixtilineus*, pro diversitate linearum quibus efficitur. Hæ lineæ (BE, BF) angulum continentes vocantur ejus *Crura*; punctum verò (B) in quo crura sibi mutuò occurrunt, est *Vertex* anguli (EBF.)



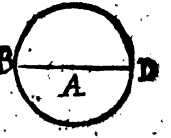
Quando angulus designatur tribus literis (EBF) media (B) denotat ipsum angulum.

14. Recta (AB) ita insistens alteri (CD) ut faciat utrinque æquales angulos (BAC, BAD) dicitur *Perpendicularis*; angulus verò à tali linea factus, est *Rectus*.



15. Angulus (FAC) minor recto (BAC) est *Acutus*; Angulus (FAD) major recto (BAD) est *obtusus*: uterque verò *Obliquus*.

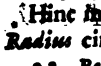
18. *Circulus* est superficies plana comprehensa circuitu unius lineæ, quæ *Circumferentia* dicitur, ad quam omnes rectæ ductæ ab aliquo puncto (A quod *Centrum* vocatur) intra circulum posito, sunt æquales.



20. *Diameter* circuli est recta (BD) quæ per centrum ducta utrinque ad circumferentiam, seu peripheriam, terminatur, & circulum bifariam dividit.

Hinc intelligitur quid sit *Semicirculus*, *Semidiameter*, seu *Radius* circuli, & *Semicircumferentia*.

23. *Rectilinea Figura* est superficies plana rectis lineis undique terminata.



24. *Triangulum* sive *Trilaterum*, est superficies plana (ABC) tribus lineis rectis (AB, BC, CA) comprehensa.

FIG. TRILATERÆ.

25. *Triang. Æquilaterum* (A) est quod habet tria latera æqualia.

26. *Isosceles* (B) est quod habet duo saltem latera æqualia.

27. *Scalenum* (C) est quod habet tria latera inæqualia.

28. *Triang. Rectangulum* est cujus unus angulus (D) est rectus.

29. *Obtusangulum* est cujus unus angulus (E) est obtusus.

30. *Acutangulum* (F) quod habet angulos tres acutos.

FIG. QUADRILATERÆ.

31. *Rectangulum* est Fig. quadril. (G vel H) habens 4 angulos rect.

32. *Quadratum* (H) est Rectangulum æquilaterum.

33. *Rhombus* (I) est, qui est æquilaterus, non æquiangulus.

34. *Rhomboides* (K) est, qui oppositа tantùm latera & angulos habet æqualia.

35. *Parallelogrammum.* (L) est cujus adversa latera sunt parallela; *Trapezium* est quævis Fig. quadril. (M *v.g.*) præter prædictas.

36. Lineæ *Parallelæ* sunt, quæ in eodem plano existentes, quantumcumque utrinque protrahantur, semper distant æqualiter. Æqualis hæc distantia mensuratur per æquales perpendiculas (AB, CD)

38. Figuræ *Multilateræ* sive *Poligonæ*, sunt quæ pluribus quàm quatuor lateribus continentur.

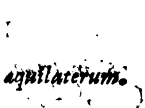
AXIO-

AXIOMATA.

1. Quæ sunt eidem tertio æqualia, sunt æqualia inter se.
2. Si æqualibus addas vel demas æqualia; manebunt æqualia.
3. Quæ sunt ejusdem vel æqualium dimidia, dupla, subtripla, tripla, &c. sunt inter se æqualia.
4. Quæ sibi mutuò congruunt, sunt æqualia.
5. Perpendicula (AB, CD) ex parallelis intercipiunt partes æquales (AC, BD.)

PROPOSITIO I.

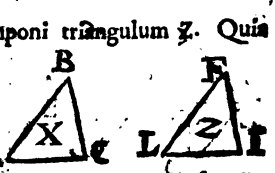
Super datâ rectâ (AB) *construere triangulum æquilaterum.*



Centris A & B, intervallo AB, describe arcus se intersecantes in C, erit triangulum ABC æquilaterum; nam latera AC, BC (*ex constr.*) æquantur lateri AB.

PROPOSITIO IV.

Si triangula duo (X & Z) *habeant circa æquales angulos* (B & F) *latera* (BA, FL, *item* BC, FI) *sibi mutuò æqualia; erunt reliqua omnia æqualia, viz. bases* (AC, LI) *& reliqui ad basim anguli* (A, L & C, I) *qui æqualibus lateribus opponuntur.*



Triangulo X concipiatur imponi triangulum Z. Quia anguli B & F sunt æquales, latera LF, FI incident in latera BA, BC; ipsisque, quia (*ex suppos.*) æqualia sunt, perfectè congruent. Ergo puncta L & I incident in A & C; adeoque basis LI congruet basi AC: ergo (*per ax.* 4) tota triangula X & Z sunt æqualia.

Scholium.

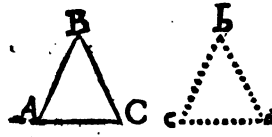
Si bases (AC, LI) *& anguli adjacentes* (A & C *ipsis* L & I) *fuerint æquales, tota triangula* (X & Z) *erunt æqualia.*

Nam si LI ponatur super AC æquale, quia æquantur anguli L & A, I & C, latus FI incidet lateri BC, & latus FL lateri BA, & punctum F in B; adeoque omnia congruent, eruntque (*per ax.* 4) æqualia.

PROPOSITIO V.

In triangulo Isoſceli anguli (A & C) *ad baſim, ſunt æquales.*

Concipe triangulum ABC iterum poni, ſed ſitu converſo,



cba. In triangulis ABC, *cba*, latera AB, *cb*, item CB, *ab*, & anguli contenti B & *b* ſunt (*ex ſuppoſ.*) æquales: ergo (*per* 4. *h*) anguli A & *c*, id eſt (quia c & C iidem ſunt) A & C ſunt æquales.

PROPOSITIO VI.

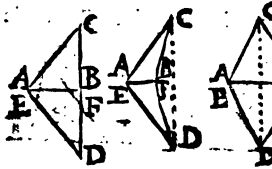
Si in triangulo anguli duo (A & C) *ſint æquales, latera* (AB, CB) *iis oppoſita ſunt æqualia.*

Convertatur triangulum ABC, ita ut fiat *cba.* In triangulis ABC, *cba*, latera AC, *ca*, & anguli adjacentes A & *c*, item C & *a* (*ex ſuppoſ.*) ſunt æquales: ergo (*per ſchol. p.* 4. *h*) reliqua omnia ſunt æqualia: ergo æquantur latera AB & *cb* ſeu CB.

PROPOSITIO VIII.

Si triangula (ABC, EFD) *ſint ſibi mutuò æquilatera, etiam anguli æqualibus lateribus oppoſiti ſunt æquales.*

Æqualia ſint latera AB & EF, AC & ED, BC & FD. Applicetur latus EF lateri æquali AB, ſed ita ut circa eundem angulum DAC ſint æqualia latera AC, ED; dein ducatur DC, & erunt (*per conſtr.*) triangula Iſoſcelia DAC; & (niſi CD coincidat cum lateribus DF, BC ut in *Fig.* 1.) DBC: ergo ſi angulis ACD, ADC (*per* 5. *h*) æqualibus addas vel ſubtrahas (ſi opus ſit) angulos BCD & BDC etiam (*per* 5. *h*) æquales; habebis angulos ACB, EDF æquales. Sed latera circa hos angulos viz: AC & ED, BC & FD (*ex ſuppoſ.*) ſunt æqualia: ergo (*per* 4. *h*) reliqua ſunt æqualia, & æquales ſunt anguli æqualibus lateribus oppoſiti.



Dixi, *niſi* CD *coincidat cum lateribus* DF, BC, *ut in Fig.* 3. quia tunc habetur unum triangulum Iſoſcele DAC, cujus

anguli D & C (*per 5. h*) sunt æquales; unde in triangulis æquilateris ACB, EDF reliqua omnia (*per 4. h*) sunt æqualia.

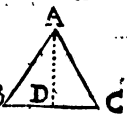
PROPOSITIO IX.

Bifariàm secare angulum datum (EAF.

Ex cruribus AE, AF accipe partes æquales AB, AC. Dein centris B & C, eodem radio, describe arcus sibi occurrentes in D: linea DA bisecabit angulum EAF. Ducantur enim BD, CD: Jam in triangulis BAD, CAD, latera AB, AC, & BD, CD sunt (*ex constr.*) æqualia, & latus AD commune: ergo (*per 8. h*) æquantur anguli BAD, CAD; adeoque bisectus est angulus EAF.

PROPOSITIO X.

Bisecare lineam datam (BC.)



Fiat (*per 1. h*) triangulum æquilaterum BAC, & (*per 9 h*) bisecetur angulus A per rectam AD, hæc bisecabit lineam BC. Nam in triangulis BAD, CAD, anguli BAD, CAD sunt (*ex constr.*) æquales, sicut & latera BA, CA; & latus AD commune: ergo (*per 4. h*) latera BD, CD sunt æqualia; adeoque bisecta est BC.

PROPOSITIO XI.

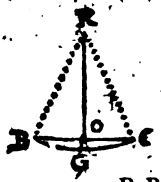
Ex puncto (O) *dato in linea recta excitare perpendiculum.*

Accipe utrinque æquales OB, OC. Dein centris B & C, eodem intervallo describe arcus se secantes in A; erit AO perpendicularis ad BC. Ducantur BA, CA.

Patet (*ex constr.*) triangula BAO, CAO esse sibi mutuò æquilatera: ergo (*per 8. h*) æquales sunt anguli AOB, AOC oppositi æqualibus lateribus BA, CA: ergo (*per 14. def. h*) AO perpendicularis est ad BC.

PROPOSITIO XII.

Ex puncto (R) *dato extra lineam* (BC) *demittere ad eam perpendiculum.*

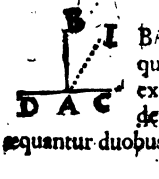


Centro R describatur arcus BGC secans rectam BC in B & C, & bisecetur BC in O; erit RO perpendicularis ad EF, Ducantur BR, CR.

Demonstratio est eadem quæ præcedentis.

PROPOSITIO XIII.

Recta insistens rectæ facit angulos æquales duobus rectis.

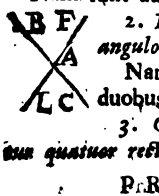


Nam si sit perpendicularis, ut BA, anguli BAD, BAC (*per 14. def. h*) sunt recti. Si obliquè insistat ut IA, angulus obtusus IAD tantùm excedit rectum BAD, quantùm acutus IAC deficit à recto BAC: ergo IAD, IAC simul æquantur duobus rectis BAD, BAC.

Corollaria.

1. *Quotcunque insistunt eidem rectæ, ad idem punctum & ad eandem partem faciunt angulos æquales duobus rectis.*

Nam sunt duo recti in plures obliquos divisi.



2. *Duæ rectæ (BC, FL) se intersecantes efficiunt angulos æquales quatuor rectis.*

Nam BAL, BAF, item CAL, CAF æquantur duobus rectis.

3. *Omnes anguli circa idem punctum æquantur quatuor rectis.* Patet ex dictis

PROPOSITIO XV.

Si duæ rectæ (BC, FL) se secuerint, anguli ad (A) verticem oppositi sunt æquales.

Nam (*per 13. h*) anguli BAL, BAF, item anguli BAL, LAC adæquant duos rectos: ergo sublato utrinque BAL, restant BAF, LAC æquales. Eodem modo ostendam BAL, FAC esse æquales.

PROPOSITIO XVIII. & XIX.

In omni triangulo angulus (A) major est qui opponitur majori lateri (OB:) & è converso.

XVIII. Ex latere majori OB aufer OF æquale lateri OA,

erit, si ducatur AF, angulus OFA (*per* 3. *Corol. p.* 32. *h*, quod ab hac non dependet) major angulo B, & (*per* 5. *h*) æqualis angulo OAF, adeoque minor angulo OAB: ergo angulus B est multò minor angulo OAB.

XIX. Converſa patet; nam si majori angulo opponetur minus latus, major angulus opponetur minori lateri; quod est contra præcedentem.

PROPOSITIO XX.

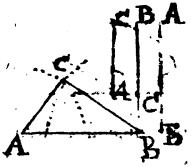
Trianguli duo quavis latera sunt reliquo majora.

Patet ex definitione lineæ rectæ.

PROPOSITIO XXII.

Ex datis tribus rectis (AB, BC, CA) *quarum duæ quævis sint reliquâ majores, triangulum constituere.*

Aſſumptâ quâlibet AB, centro B, intervallo datæ BC deſcribe arcum; & centro A, intervallo alterius datæ AC deſcribe arcum priori occurrentem in C. Ductis AC, BC, erit ABC triangulum quæſitum. Patet *ex conſtr.*



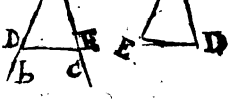
Corollarium.

Eodem modo fiet triangulum æquale cuicunque dato.

PROPOSITIO XXIII.

Ad datum punctum (L) *angulum efficere æqualem dato* (A.)

Ducatur utcunque DE Secans crura anguli A, & triangulo ADE conſtitue (*per* 22. *h*) æquale LDE; eritque angulus L (*per* 8. *h*) æqualis dato (A.)



Scholium.

Quia in Operationibus Mathematicis ſæpe requiruntur anguli, vel pares datis, vel certum graduum numerum continentes; pro angulorum praxi aliqua ſubjungam.

1. *Ad datum punctum* (L) *ſtatuere angulum parem dato* (A.) Centro A deſcribe inter anguli dati latera arcum DE; & centro

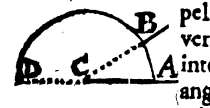
centro L, eodem radio describe DF. Ex DF aufer DE æqualem alteri DE, & per E duc rectas LD, LE. Erit ELD par dato A.

2. *Anguli dati* (A) *gradus examinare.* Inter crura anguli A, centro A, describe arcum DE; quem circino exceptum applica quadranti vel semicirculo in gradus divisо. Notæ margini adscriptæ quantitatem anguli indicabunt.

3. *Angulum construere continentem gradus datos.* Centro quovis L describatur arcus DF; ex quo si major sit, auferatur DE par arcui quadrantis continenti gradus datos, & ducantur rectæ LE, LD. Erit ELD angulus quæsitus.

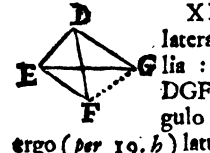
Nota, arcus DE radium debere in utroque casu æqualem esse semidiametro alicujus circuli instrumento inscripti.

Si ad manum sit semicirculus excavatus, vel ex materia pellucida, satis erit ejus centrum super C verticem anguli collocare; nam arcus AB inter crura contentus ostendet quantitatem anguli dati ACB, sicut & puncta B & A, per quæ à centro ductæ CB, CA efficient angulum BCA continentem gradus quæsitos.



PROPOSITIO XXIV. & XXV.

Si duo triangula (EDG, EDF) *habeant circa angulos inæquales* (EDG, EDF) *latera* (DE, DF & DE, DG) *sibi mutuò æqualia; erit basis* (EG) *oppositа majori angulo* (EDG) *majori basi* (EF) *oppositæ minori angulo* (EDF:) *& è converso.*



XXIV. Ducatur FG. In triangulo FDG latera DF, DG sunt (*ex suppositione.*) æqualia: ergo (*per 5. h*) æquales sunt anguli DGF, DFG: ergo angulus EGF minor est angulo DFG, multòque minor angulo EFG: ergo (*per 19. h*) latus EG majus est latere EF.

XXV. Si anguli EDF, EDG essent æquales; bases EF, EG (*per 4. h*) essent æquales. Si EDG esset minor, basis EG (*per præced.*) esset minor. Utrumque verò est contra hypothesim, nam EG supponitur major quàm EF. Ergo angulus EDG est major quàm EDF.

PROPOSITIO XXVI.

Si duo triangula (X & Z. Vide Fig. Prop. 4.) *habeant duos angulos duobus æquales* (A *ipsi* L, & B *ipsi* F) *alterum alteri, unumque latus uni lateri æquale, vel quod existit inter æquales angulos* (*ut* AB, LF) *vel quod eorum uni opponitur* (*ut* BC, FI) *omnia reliqua erunt æqualia.*

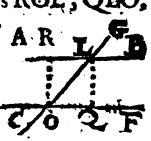
Nam (*per* 5. *Corol. p.* 32. *h.* quod ab hac non dependet) tertius angulus C æquatur tertio I. Ergo (*per Schol. p.* 4. *h.*) reliqua omnia sunt æqualia.

PROPOSITIO XXVII.

Si rectas (AB, CF) *parallelas secet recta* (GO.)

1.° *Æquales sunt anguli alterni* (ALO, FOL, *item* BLO, COL.)

Duc perpendicula OR, LQ. In triangulis ROL, QLO, latus LO est commune; latera RO, LQ [*per* 36. *def. h*] & latera RL, OQ [*per ax.* 5] sunt æqualia: ergo [*per* 8. *h*] æquantur anguli alterni RLO, QOL; sicut & anguli ROL, QLO; quibus si addantur COR, QLB recti, adeoque æquales, æquales erunt etiam anguli alterni COL, BLO.



2.° *Externus* [GLB] *æquatur interno ad eandem partem* [LOQ.]

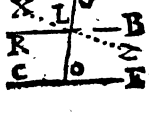
Nam [*per* 15. *h*] GLB æquatur angulo RLO, id est [*per* 1. *partem*] angulo LOQ.

3.° *Interni ad eandem partem* [BLO, FOL, *item* RLO, COL] *æquantur duobus rectis.*

Nam [*per* 13. *h*] BLO cum RLO, id est [*per* 1. *partem*] cum FOL, adæquat duos rectos.

PROPOSITIO XXVIII. & XXIX.

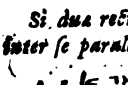
Si rectas [RB, CE *secans recta* [GO] *faciat aut angulos alternos* [RLO, EOL] *æquales; aut externum* [GLB] *æqualem interno ad eandem partem* [LOE] *aut duos internos ad eandem partem* [BLO, EOL] *æquales duobus rectis; recta illæ* [RB, CE] *sunt parallelæ.*



Si neges; fit XLZ parallela ad CE: ergo [*per 17. h*] XLZ cum GO, CE facit angulos omnes, quales descripsimus; *id est* [*ex suppos.*] eosdem quos cum iisdem GO, CE facit RB. Quod est absurdum: ergo RB est parallela ad CE.

PROPOSITIO XXX.

Si duæ rectæ [AB, CF] *sint parallelæ eidem* [DN,] *sunt inter se parallelæ.*



Nam si ducatur recta GO, erit [*per 27. h*] externus GLB æqualis interno LDN, & LDN [*per eandem*] interno DOF: ergo externus GLB [*per ax. 1.*] æquatur interno DOF seu LOF: ergo [*per 29. h*] parallelæ sunt AB, CF.

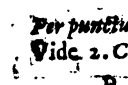
PROPOSITIO XXXI.

Per punctum datum ducere parallelam ad rectam datam.

Vide 2. *Coroll. p. 29. l. 3.* quod ab hac non dependet.

PROPOSITIO XXXII.

In omni triangulo rectilineo angulus quilibet externus (FBC) *æquatur internis oppositis* (A & C) *similiter sumptis; & omnes tres anguli interni* (A, B, C) *simul conficiunt duos rectos.*



1. *Pars.* Fiat BL parallela ad AC. Angulus externus FBL (*per 27. h*) æquatur interno A, & alterni LBC, BCA [*per eandem*] sunt æquales. Sed FBL, LBC sunt angulus externus FBC: ergo FBC æquatur internis oppositis A & C.

2. *Pars.* Angulus ABC cum FBC (*per 13. h*) æquatur duobus rectis. Sed FBC (*per 1. pantem*) æquatur angulis A & C: ergo ABC cum A & C facit duos rectos.

Corollaria.

1. *Si in triangulo quovis* (ABC) *sit aliquis angulus* [C] *rectus, reliqui* (A & B) *sunt acuti, & simul æquales uni recto.*

Nam aliter A & B cum C non conficerent duos rectos.

2. *Si angulus aliquis sit obtusus, reliqui duo sunt acuti.*

Nam aliter omnes simul excederent duos rectos. 3. An-

3. *Angulus externus major est quolibet internorum oppositorum.*

Nam æquatur utrique simul.

4. *Noto uno angulo, cognoscitur Summa reliquorum: & notâ duorum Summâ, cognoscitur tertius.*

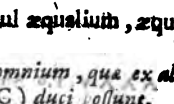
Scitur enim, quantum noti desit ad duos rectos.

5. *Cum in uno triangulo duo anguli aut simul, aut singuli, æquales sunt duobus aut simul, aut singulis in altero triangulo; etiam tertius tertio æqualis erit.*

Nam angularum vel seorsim, vel simul æqualium, æqualis est differentia à duobus rectis.

6. *Perpendicularis* (AD) *est brevissima omnium, quæ ex aliquo puncto* (A) *ad rectam quamlibet* (BC) *duci possunt.*

Nam quomodocunque ducatur AD, erit angulus D rectus, adeoque B acutus: ergo (*per* 19. *h*) latus AB excedit latus AD.



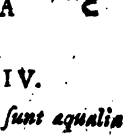
7. *Si ab uno puncto* (A) *in rectam* (EC) *cadant duæ rectæ, altera* (AB) *obliquè, altera* (AD) *perpendiculariter; hæc* (AD) *cadet versùs partes anguli acuti* (ABC.)

Nam si caderet versùs obtusum ABE, in E *v. g.* essent in triangulo AEB, angulus E (*per* 14. *def. h*) rectus, & ABE (*ex suppos.*) obtusus, quod repugnat 1. *Corol.*

PROPOSITIO XXXIII.

Si æquales parallelas (AB, CF) *jungant rectæ* (AC, BF) *erunt hæ æquales & parallelæ.*

Ducatur AF. In triangulis BAF, CAF, æquales sunt (*per* 27. *h*) anguli alterni BAF, CFA; sicut & (*ex suppos.*) latera BA, CF, & latus AF est commune: ergo (*per* 4. *h*) æqualia sunt latera AC, BF; & anguli CAF, BFA: ergo (*per* 28. *h*) latera AC, BF sunt etiam parallela.



PROPOSITIO XXXIV.

Parallelogrammi oppositi anguli & latera sunt æqualia, ipsumque à diametro bisecatur.

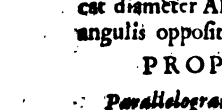
1.° Ductâ diametro AF, æquales sunt (*per* 27. *h*) anguli

alterni BFA, CAF, item BAF, CFA: ergo totus BAC æquatur toti BFC. Similiter oftendam angulos B & C esse æquales. 2.° Quia ad basim communem AF æquales sunt anguli BAF, CFA, & BFA, CAF; tota triangula ABF, ACF (*per schol. p. 4. h.*) sunt æqualia; adeoque parallelogrammum bifecat diameter AF; & latera BF, AC, & BA, FC, æqualibus angulis opposita sunt æqualia.

PROPOSITIO XXXV. & XXXVI.

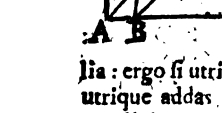
Parallelogramma super eadem vel æquali basi & inter easdem parallelas posita, sunt æqualia.

XXXV. 1.° Sint parallelogramma (ACDB, ALQB habentia eandem basim AB. In triangulis ALC, BQD, latera AC,



DB, & AL, BQ (*per 34. h.*) sunt æqualia: & (*per 27. h.*) anguli comprehensi CAL, DBQ æquantur angulo externo DOL, adeoque (*per ax. 1.*) sunt inter se æquales: ergo [*per 4. h.*) tota triangula ALC, BQD sunt æqualia: ergo si utrique auferas triangulum commune DOL, & utrique addas triangulum AOB, erunt (*per ax. 2.*) æqualia parallelogramma ABDC, ALQB.

XXXVI. Si habeant bases distinctas sed æquales AB, GH;



ducantur GC, HD. Jam quia AB, GH sunt (*ex suppos.*) & AB, CD (*per 34. h.*) æquales; GH, CD sunt (*per ax. 1.*) æquales. Sed (*ex suppos.*) sunt etiam parallelæ: ergo (*per 33. h.*) GC, HD sunt æquales & parallelæ; adeoque GCDH (*per 35. def. h.*) est parallelogrammum, cui (*per 35. h.*) æquantur parallelogramma ACDB, GEFH: ergo hæc sunt (*per ax. 1.*) inter se æqualia.

Corollarium.

Efficere rectangulum æquale parallelogrammo dato [ACDB.]

Acceptâ basi æquali GH, erige perpendicula GE, HF occurrentia lateri CD producto. Erunt [*per 36. h.*] ACDB, GEFH æqualia.

PROPOSITIO XXXVII. & XXXVIII.

Triangula (ACB, AFB) *super eadem basi* (AB) *vel aequalis & inter easdem parallelas constituta sunt aequalia.*

XXXVII. Ducantur BL, BI parallelæ lateribus AC, AF. Parallelogramma ACLB, AFIB (*per 35. h*) sunt æqualia. Sed eorum dimidia sunt (*per 34. h*) triangula ACB, AFB: ergo (*per ax. 3.*) hæc sunt æqualia.

XXXVIII. Si triangula habeant æquales tantùm bases; eodem modo ostendentur esse dimidia parallelogrammorum iisdem basibus æqualibus insistentium. Sed hæc (*per 36. h*) sunt æqualia: ergo (*per ax. 3.*) ipsa triangula sunt æqualia.

PROPOSITIO XLI.

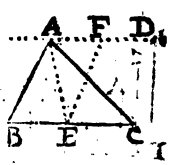
Si triangulum (AFB) *sit in iisdem parallelis & eâdem vel æquali basi* (AB) *cum parallelogrammo* (LA); *erit ipsius dimidium.*

Ducatur CB triangulum AFB (*per 37. h*) æquatur triangulo ACB; id est (*per 34. h*) dimidio parallelogrammi AL.

PROPOSITIO XLII.

Construere parallelogrammum (ABEF) *æquale triangulo dato* [ABC.]

Trianguli dati basim BC bisecta in E; dein per anguli oppositi verticem A duc AD parallelam basi BC, & per E duc EF parallelam alterutri lateri AB. Eritque parallelogrammum ABEF æquale triangulo dato ABC.

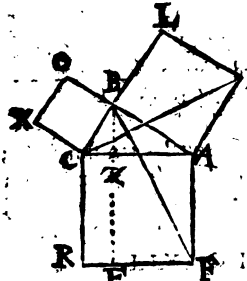


Ducatur AE; eruntque triangula ABE, AEC (*per 38. h*) æqualia: ergo ABE est dimidium trianguli ABC. Sed (*per. 41. h*) ABE est dimidium parallelogrammi ABEF: ergo ABC, ABEF (*per ax. 3.*) sunt æqualia.

PROPOSITIO XLVII.

In omni triangulo rectangulo quadratum [CAFR *lateris* (AC) *oppositi angulo recto* [B] *æquatur reliquorum laterum* (AB, CB) *quadratis* [AILB, BOXC] *simul sumptis.*

Ducantur IC & FB cum BE parallelâ ad AF. Jam si angulis IAB, FAC (*per 32. def. h.*) rectis, adeoque æqualibus addatur angulus BAC, erunt (*per ax. 2.*) anguli IAC, FAB æquales. Sed etiam latera IA, BA, & CA, FA, hos angulos continentia, sunt (*per 32. def. h.*) æqualia: ergo (*per 4. h.*) triangula IAC, FAB sunt æqualia. Sed (*per 41. h.*) hæc triangula sunt dimidia rectangulorum BAIL, ZAFE: ergo [*per ax. 3.*] rectangula BAIL, ZAFE sunt æqualia. Eodem modo ostendam rectangula BCXO, ZERC esse æqualia: ergo rectangula ZAFE, ZERC, id est, quadratum AC æquatur rectangulis BAIL, BCXO, id est, quadratis AB, CB.



Corollaria.

1. *Datis pluribus quadratis, unum omnibus æquale construere.*

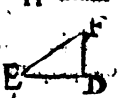
Sint quadratorum latera AB, CD, EF. Fiat angulus rectus X habens latera infinita SX, XZ. Lateri XS applica lineam AB ab X ad B, & lineam CD lateri XZ ab X ad D: erit quadratum lateris DB æquale quadratis XB, XD, id est AB, CD. Rursus EF applica lateri XZ ab X ad F; & DB lateri XS ab X ad C, erit quadratum FC æquale quadratis FX & XC, id est quadratis AB, CD, EF datis.

2. *Datis duobus quadratis* (EF, CD) *eorum differentiam invenire.*

Quadrati minoris latus CD applica lineæ XZ ab X ad D; & quadrati majoris latus EF ex puncto D extende ad lineam SX, & pertingat à D ad B *v.g.* erit quadratum XB differentia quadratorum XD, DB, seu CD, EF.

3. *Notis duobus quibuscunque lateribus trianguli rectanguli, tertium invenire.*

1.° Si cognoscatur hypotenusa, seu latus EF oppositum angulo recto D, cum alio aliquo latere FD; huic lateri FD quadratum subtractum ex quadrato hypotenusæ EF, relinquet quadratum lateris DE; cujus si radicem extraxeris, notum erit latus ED.



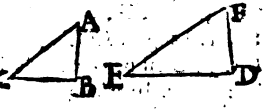
2.° Si dentur duo latera ED, FD comprehendentia angulum rectum D; eorum quadrata in summam collecta dabunt quadratum hypotenusæ EF.

4. *Si duo triangula rectangula* (ABC, FKI) *habeant hypotenusas* (AB, FK) *& latera quævis* (BC, KI) *æqualia, tertium* (AC) *æquatur tertio* (FI.)

Nam si quadratis æqualibus AB, FK, subtrahantur æqualia quadrata BC, KI; relinquentur quadrata, adeoque & latera æqualia AC, FI.

5. *Si in triangulis rectangulis* (ABC, FDE) *sint circa angulos rectos* (B & D) *latus unum* (AB) *æquale uni* (FD) *& aliud* (BC) *minus alio* (DE;) *erit unius hypotenusa* (AC) *minor alia* (EF.)

Nam quia quadrata FD, DE simul excedunt quadrata AB, CB; quadratum FE excedit quadratum AC, adeoque & recta FE excedit rectam AC.



EX LIBRO TERTIO.

DEFINITIONES.

2. Recta (AB) circulum tangit quæ illi in occurrit in (B) & producta illum non secet.

Subtensa arcûs (EL) est recta connectens ejus extrema (E & L.)

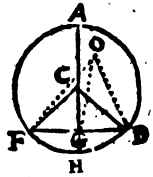
5. Segmenta circuli sunt partes, in quas eum dividit secans (EL.)

6. Angulus in segmento est (EFL) qui continetur sub rectis lineis (EF, LF) quæ ad (F) unum circumferentiæ punctum ex segmenti terminis (E, L) ducuntur.

PROPOSITIO I.

Circuli dati (FAD) *centrum invenire.*

Subtensam FD utcunque ductam biseca in G, & per G duc perpendicularem AH, quam bisecta in C. Erit C centrum.



Si neges; sit centrum O extra rectam AH (nam nequit esse in AH, cùm ubilibet extra C dividatur inæqualiter) & ducantur OF, OG, OD. In triangulis OGF, OGD latera FG, DG (*ex constr.*) & OF, OD (*per* 18. *def. l.* 1.) sunt æqualia; & OG commune: ergo (*per* 8. *l.* 1.) anguli OGF, OGD sunt æquales, adeoque (*per* 14. *def. l.* 1.) recti; quod absurdum est, cùm AGF, AGD sint (*ex constr.*) recti.

PROPOSITIO III.

Si recta (AH) *per centrum ducta aliam* (FD) *non per centrum ductam secet bifariam, secat perpendiculariter: & è converso.*

1. *Pars.* Ducantur radii CF, CD; qui cùm æquales sint, sicut & [*ex suppos.*] latera FG, DG, & latus CG commune; triangula CFG, CDG sunt sibi mutuò æquilatera: ergo [*per* 8. *l.* 1.] anguli CGF, CGD sunt æquales: ergo [*per* 14. *def. l.* 1.] CG est perpendicularis ad FD.

2. *Pars.* In triangulis CGF, CGD, anguli ad G sunt [*ex suppos.*] recti; hypotenuæ CF, CD [*per* 18. *def. l.* 1.] æquales; & latus CG commune: ergo [*per* 4. *Corol. p.* 47. *l.* 1.] latera FG, DG sunt æqualia.

Corollaria.

1. *Datum arcum* (FHD) *bisecare.*

Per subtensam FD bisectam in G, due perpendicularem GH; hæc bisecabit arcum datum in H.

Nam HG producta versùs A transibit (ut ostensum est *in* 1. *h*) per centrum C, ex quo si ducantur radii CF, CD; ostenderur (*ut in* 3. *h*) angulos FCG, GCD, adeoque & [*per* 28. *h.* quæ ab hac non dependet] arcus FH, HD esse æquales.

2. *Perficere arcum datum* [ABC.]

Rectas AB, BC utcunque subtensas biseca in L & I. Per L

& I duc perpendiculares LE, IF, sibi occurrentes in P. Erit P centrum circuli.

Nam (*ex dicto*) centrum est tam in LE, quàm in IF; adeoque in communi puncto P.

3. *Describere circulum per tria puncta data* (A, B, C) *non in directum posita.*

Ducantur rectæ AB, BC; quibus bisectis in L & I, excitentur perpendiculares LE, IF concurrentes in P. Circulus ductus centro P, radio PC, transibit per puncta data.

Nam, ductis PA, PB, PC, erunt in triangulis PIB, PIC (*ex constr.*) latera BI, CI, æqualia, PI commune, & anguli PIB, PIC recti, adeoque æquales: ergo (*per 4. l. 1.*) æquantur latera PB, PC; & (quod eodem modo ostenditur) PB, PA. Ergo circulus descriptus per C, transit etiam per B & A.

PROPOSITIO V. & VI.

Circuli se mutuò secantes vel tangentes non habent idem centrum.

Patet, quòd si sint concentrici, circulus cujus radius est major, totus cadet extra alium; nam distantia peripheriæ à centro ubique æquatur radio.

PROPOSITIO VII.

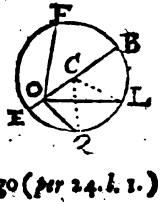
Si in circulo ex puncto (O) *quod non est centrum, plures rectæ* [OB, OL, OQ, OE] *in circumferentiam cadant;*

1.° *Maxima erit* (OB) *per* (C) *centrum transiens.*

Nam ductâ CL, patet [*ex 20. l. 1.*] quòd OC, CL, id est [quia æquales sunt CB, CL] quòd OB major sit quàm OL.

2.° *Reliqua diametri pars* [OE] *est minima.*

Nam ductâ CQ, patet quòd CO, OQ simul excedant CQ, id est CE: ergo sublato utrinque CO, latus OQ excedit OE.



3.° *Aliarum major est, quæ maxima* (OB) *propior.*

Nam in triangulis COQ, CLO, latera LC, CO æquantur lateribus QC, CO: angulus verò LCO major est quàm QCO: ergo (*per 24. l. 1.*) basis OL excedit basim OQ.

4.o *Ex O quod non est centrum, non plures quàm duæ ad circumferentiam duci possunt æquales.*

Patet; nam si ducerentur tres æquales OL, OF, OQ, essent ad eandem partem duæ æquales OL, OQ; quod repugnat 3. *parti.*

PROPOSITIO IX.

Si ab aliquo intra circulum puncto plures quàm duæ ad ambitum duci possunt æquales, illud punctum est centrum.

Patet *ex* 4. *parte præcedentis.*

PROPOSITIO X.

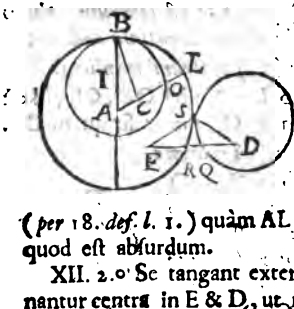
Circuli in duobus tantùm punctis se mutuò secant.

Si se mutuò secuerint in tribus; rectæ ex centro utriusvis circuli ductæ ad illa tria puncta, erunt (*per* 18. *def. l.* 1.) æquales: ergo (*per* 9. *h*) centrum illud est utrique circulo commune: ergo circuli concentrici se mutuò secant; quod est contra quintam *h*.

PROPOSITIO XI. & XII.

Si duo circuli se quomodocunque tangant, recta centrum centra connectens per contactum transit.

XI. 1.o Se tangant interiùs in B. Si AB ducta ex A centro circuli BL ad contactum B non transeat per centrum circuli BO, erit centrum illud alicubi extra lineam AB, *v. g.* in C. Ducantur per C rectæ AL, BC. Quia C est centrum, CB; CO sunt æquales; & additâ utrique AC, erunt AC, CB æquales AO: sed AC, CB sunt majores (*per* 20. *l.* 1.) quàm AB hoc est (*per* 18. *def. l.* 1.) quàm AL: ergo AO major est quàm AL; quod est absurdum.



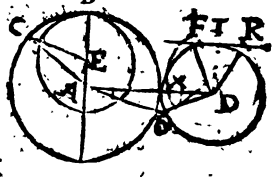
XII. 2.o Se tangant exteriùs in S, & si fieri potest, ita ponantur centra in E & D, ut recta ED non transeat per contactum S. Quia E & D sunt centra, radii ES, ER, item DQ, DS sunt æquales: ergo tota ED major est quàm ES & DS, quod repugnat vigesimæ *l. primi.*

PRO-

PROPOSITIO XIII.

Circuli se mutuò & lineam rectam punctualiter tangunt.

1.° Si fieri potest, tangant se interiùs in duobus punctis B & C. Recta AE ducta per centra A & E (*per* 11. *h*) transit per contactum, puta per B: ducantur etiam AC, EC. Quia E est centrum, EB, EC sunt æquales: ergo additâ utrique AE, rectæ AE, EC æquales sunt radio AB, seu AC, quod est contra vigesimam *l.* 1.



2.° Tangant se exteriùs in X & O. Erunt radii AX, DX, seu (*per* 12. *h*) recta AD, æquales radiis AO, DO, quod est contra vigesimam *l.* 1.

3.° Recta FR tangat circulum in F & R; erunt radii DF, DR æquales: ergo (*per* 5. *l.* 1.) anguli DFR, DRF sunt æquales, adeoque acuti: ergo DI ducta perpendiculariter à centro D ad rectam FR (*per* 6. & 7. *Corol. p.* 32. *l.* 1) cadet inter F & R, eritque minor radio DR: ergo punctum I est intra circulum: ergo FR secat circulum.

PROPOSITIO XVI.

Recta (AB) quæ perpendicularis est ad diametri extremitatem (A), tota cadit extra circulum.

Ad rectæ AB quodvis punctum B ducatur à centro recta CB. Quia angulus CAB ponitur rectus, CBA [*per* 1. *Corol. p.* 32. *l.* 1.] est acutus: ergo (*per* 19. *l.* 1.) CB major quàm CA: ergo punctum B, & (ob eandem rationem) omnia puncta lineæ AB, cadunt extra circulum.

PROPOSITIO XVII.

Ex puncto dato rectam ducere, quæ tangat circulum datum. Vide *Corol. p.* 31. *h.*

PROPOSITIO XVIII.

Recta [CA] quæ ducitur ex centro ad circulum [in A] & tangenti (AB) perpendicularis.

Si neges; sit CB perpendicularis; erit angulus B rectus; adeoque [*per* 1. *Corol. p.* 32. *l.* 1.] A acutus: ergo (*per* 19. *l.* 1.) CB minor est quàm CA, id est (*per* 18. *def. l.* 1.) quàm CD, quod est absurdum.

PROPOSITIO XX.

Angulus (ACD) *ad centrum duplus est anguli* (AFD) *ad peripheriam insistentis idem arcui* (AD.)

1.° Coincidant AC, AF, erit (*per* 32. *l.* 1.) angulus ACD æqualis angulis D & F; id est, quia CD, CF, adeoque (*per* 5. *l.* 1.) anguli D & F sunt æquales, duplus anguli F.

2.° Si AC, DC cadant inter AF, DF, ducatur diameter FE; & erit (*ut priùs*) ECA duplus anguli EFA, & ECD duplus anguli EFD: ergo totus ACD est duplus totius AFD.

3.° Si unum latus AF secet aliud DC, ducatur diameter FE; & erit (*ut priùs*) ECD duplus anguli EFD, & ECA duplus anguli EFA: ergo sublatis ECA, EFA, restat ACD duplus anguli AFD.

PROPOSITIO XXI.

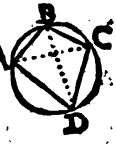
Anguli (ADB) *ad peripheriam, qui in circulo insistunt eidem arcui, vel sunt in eodem segmento, sunt æquales.*

1.° Si segmentum ADB sit majus semicirculo, angulus ad centrum ACB (*per* 20. *h*) est duplus singulorum ADB: ergo (*per ax.* 3.) anguli ADB sunt inter se æquales.

2.° Si segmentum ADB minus sit semicirculo, vel æquale; in triangulis DAT, DBT, anguli ad T (*per* 15. *l.* 1.) & anguli A & B (*per* 1. *partem h*) sunt æquales: ergo [*per* 5. *Corol. p.* 32. *l.* 1.] angulus tertius D æquatur tertio D.

PROPOSITIO XXII.

Quadrilateri (ABCD) *circulo inſcripti quivis duo anguli oppoſiti* (A & C, B & D) *ſimul faciunt duos rectos.*



Ducantur AC, BD. Angulus ABC cum BCA, & BAC (*per* 32. *l.* 1.) facit duos rectos: ſed (*per* 21. *b*) anguli BCA, BDA, & BAC, BDC ſunt æquales: ergo ABC cum BDA, BDC, id eſt cum ADC facit duos rectos.

PROPOSITIO XXV.

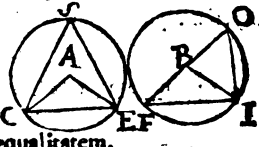
Datum arcum perficere.

Vide 2. *Conol. p.* 3. *b.*

PROPOSITIO XXVI. & XXVII.

In æqualibus circulis recta æquales (CE, FI) *ſubtendunt arcus æquales; & ſi arcus ſint æquales, ſubtenſæ ſunt æquales.*

XXVI. Ducantur radii AC, AE, BF, BI; & fient triangula ACE, BFI ſibi mutuò æquilatera, adeoque (*per* 8. *l.* 1.) anguli A & B æquales: ergo crura BF, BI impoſita cruribus AC, AE, ipſis congruent; ſicut & arcus IF arcui CE ob circulorum æqualitatem.



XXVII. Quia arcus CE, FI ponuntur æquales, ſibi mutuò impoſiti congruent, & puncta F, I cadent in puncta C, E: ergo ſubtenſæ ſibi mutuò congruent; adeoque [*per ax.* 4.] erunt æquales.

Corollarium.

Inter arcus (CE, IO) *ſemicirculo minores, ille* (CE) *eſt major, cujus major eſt ſubtenſa.*

Nam ſi ducantur radii AC, AE, BI, BO; erunt (*per* 18. *def. l.* 1.) in triangulis CAE, IBO, latera AC, BI, & AE, BO æqualia, & baſis CE (*ex ſuppoſ.*) major baſi IO: ergo (*per* 25. *l.* 1.) angulus CAE major eſt angulo IBO: ergo ſi latus BI imponatur lateri AE, latus BO cadet inter C & E, & arcus IO non pertinget à C ad E; adeoque arcus IO minor eſt arcu CE.

PROPOSITIO XXVIII. & XXIX.

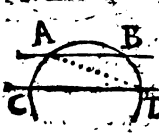
In circulis æqualibus ſi anguli, ſive ad centra, ſive ad ambitum ſint æquales; arcus, quibus inſiſtunt, ſunt æquales: & è converſo.

XXVIII. 1. *Pars.* Quia in triangulis ACE, BFI, latera AC, BF, & AE, BI, & anguli contenti A & B ſunt (*ex ſuppoſ.*) æquales, æquantur (*per* 4. *l.* 1.) latera CE, FI; adeoque (*per* 27. *h*) etiam arcus CE, FI.

2. *Pars.* Anguli CAE, FBI ſunt (*per* 20. *h*) dupli angulorum CSE, FOI: ergo (*per ax.* 3.) ſi anguli CSE, FOI ſint æquales, æquantur anguli CAE, FBI; adeoque (*per* 1. *partem*) etiam arcus CE, FI.

XXIX. Quia arcus CE, FI ponuntur æquales, æquantur (*per* 27. *h*) ſubtenſæ CE, FI; adeoque triangula ACE, BFI ſunt ſibi mutuò æquilatera; ergo (*per* 8. *l.* 1.) æquales ſunt anguli A & B: quorum dimidia ſunt (*per* 20. *h*) anguli S & O, etiam ipſi (*per ax.* 3.) ſunt æquales.

Corollaria.



1. Si rectæ AB, CD intercipiant arcus æquales AC, BD; ſunt parallelæ. Nam ducta AD, anguli alterni BAD, CDA [*per* 29. *h*] ſunt æquales ergo [*per* 28. *l.* 1.] AB, CD ſunt parallelæ.

2. Si per datum punctum D oporteat parallelam ducere ad rectam datam AB; quævis centro ducatur circulus tranſiens per D, & ſecans ubicunque datam AB in B & A, arcui BD accipe æqualem AC, recta CD erit parallela datæ AB

PROPOSITIO XXX.

Datum arcum biſecare.

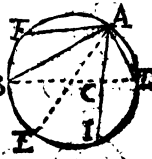
Vide 1. *Corol. p.* 3. *h.*

PROPOSITIO XXXI.

Angulus (BAD) *in ſemicirculo eſt rectus; in ſegmento autem eſt minor recto, & minor eſt major recto.*

1. *Pars.* Ducantur diametri AE, BD; eritque (*per* 20. *h*)

angulus BCE duplus anguli BAE, & angulus ECD duplus anguli EAD. Sed BCE, ECD simul [*per* 13. *l.* 1.] æquantur duobus rectis: ergo BAE, EAD simul, id est BAD æquatur uni recto.

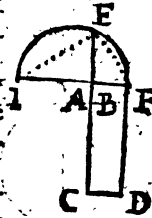


2. & 3. *Pars.* Patet quòd, quomodocunque ducantur IA, & FA, erunt in segmento majore angulus IAD minor, & in segmento minore, angulus FAD major recto BAD.

Corollaria.

1. *Quadratum exhibere æquale cuicunque Figuræ rectilineæ data.*

1.° Proponatur rectangulum BCDF. Latus minus FB productum in I, donec æquetur majori BC, bisecta in A; & centro A per I & F describe semicirculum, cui in E occurrat CB productum. Demum duc rectas IE, EF; eritque angulus IEF rectus, & quadratum EF æquale rectangulo BCDF. Demonstratur ut Prop. 47. *l.* 1.



2.° Si detur parallelogrammum obliquangulum fiat (*per Corol. p.* 36. *l.* 1.) rectangulum ei æquale, & (*per* 1. *partem*) quadratum æquale huic rectangulo.

3.° Si detur triangulum, fiat (*per* 42. *l.* 1.) ei æquale parallelogrammum, & (*per* 1. & 2. *partem*) huic æquale quadratum.

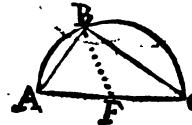
4.° Si proponatur alia quævis Figura, resolvatur in triangula; quibus (*per* 3. *partem*) fiant totidem quadrata æqualia; & his omnibus (*per* 1. *Corol. p.* 47. *l.* 1.) unum quadratum æquale.

2. *Ex dato puncto* (A) *rectam ducere, quæ tangat circulum datum.*

Ab A ad circuli dati centrum C ducatur recta AC; quâ bisectâ in B, centro B, radio BA, describatur semicirculus occurrens circulo dato in D. Recta AD tanget circulum datum.

Nam, ductâ CD, angulus ADC in semicirculo [*per* 31. *h*] est rectus: ergo [*per* 17. *h*] AD tangit circulum.

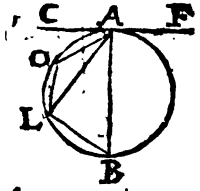
3. *Ad lineæ datæ extremum punctum* [B] *ducere perpendiculum* (AB.)



Accepto utcunque centro F extra datam BC, radio FB describatur circulus ABC secans datam alicubi in C; ductâque diametro CA, jungantur A & B. Erit AB perpendicularis ad BC in B; nam (*per* 31. *h*) angulus ABC est rectus.

PROPOSITIO XXXII.

Si recta (CF) *circulum tangat, & alia* (AB, *vel* AL, *vel* AO) *ex* (A) *contactu ducta eundem secet; erit angulus à tangente & secante factus, par angulo qui fit in segmento alterno id est, æquales erunt anguli* FAB, ALB, & CAL, ABL, & FAL, AOL.



1.° Si secans AB transeat per centrum, recti sunt anguli, FAB (*per* 18. *h*) & ALB (*per* 31. *h*) adeoque æquales.

2.° Si secans AL per centrum non transeat; ducatur per centrum recta AB, & jungantur L & B. Quia angulus ALB (*per* 31. *h*) est rectus, anguli LBA, LAB (*per* 1. *Corol. p.* 32. *l.* 1.) simul efficiunt unum rectum. Sed (*per* 18. *h*) CAL, LAB simul, seu CAB æquatur uni recto: ergo ablato communi LAB, restat CAL æqualis angulo LBA in segmento alterno.

3.° Quadrilateri AOLB anguli oppositi O & B (*per* 22. *h*) simul adæquant duos rectos. Sed LAC, LAF (*per* 13. *l.* 1.) simul constituunt duos rectos: ergo subductis LAC, LBA (*per* 2. *partem*) æqualibus, restat LAF æqualis angulo LOA in segmento alterno.

PROPOSITIO XXXV.

Si duæ rectæ se in circulo secuerint, rectangulum sub segmentis unius, æquale est rectangulo sub segmentis alterius.

Vide *Corol. p.* 16. *l.* 6.

PROPOSITIO XXXVI.

Si ex puncto extra circulum una recta tangat, alia secet circulum; erit quadratum tangentis æquale rectangulo sub tota secante & parte inter punctum & circulum interjectâ.

Vide 1. *Corol. p.* 17. *l.* 6. ubi etiam duo hujus Propositionis Corollaria demonstrantur.

EX LIBRO QUINTO.

Quia Libri hujus Theoremata, non solùm lineis, sed omni omninò quantitati conveniunt, ad terminos exprimendos Alphabeti literas adhibemus, quæ Propositionum universalitatem meliùs indicant. Brevitatis etiam & claritatis causâ utimur notis quibusdam, apud Geometras receptissimis; quas ne Tyronem morentur, breviter explico.

1.o Proportio sic exprimitur A : B :: C : D. Lege A est ad B, ut C ad D.

2.o = Significat æqualitatem. Ut A = , id est quantitas A æqualis est quantitati B.

3.o + Est signum Additionis. Sic A + B significat A plus B, seu A cum B.

4.o — Denotat subtractionem. Sic A — B significat A minùs B, seu A mulctatam quantitatem B.

Propositiones aliquæ hujus Libri sunt per se ita manifestæ, ut probatione non indigeant; quam proinde omisimus, ne (quod interdum contingit) res per se notæ demonstrando fierent obscuræ.

DEFINITIONES.

3. *Ratio* est duarum ejusdem generis magnitudinum mutua secundùm quantitatem habitudo. Quantitas quæ comparatur, dicitur *Antecedens*: illa, cui comparatur *Consequens*.

5 Rationes sunt *similes*, *Æquales*, *Eadem*, quarum Antecedentia eandem habent magnitudinem respectu suorum Consequentium; ut si A sit semissis vel duplum B, C semissis vel duplum D.

Proportio est rationum similitudo, æqualitas, vel identitas.

8. Datur *Proportio continua*, quando terminus qui est Con-

sequens respectu unius Antecedentis, est Antecedens respectu alterius Consequentis; seu quando medii termini bis sumuntur; ut A : B :: B : C. Datur *Proportio discreta*, cùm nullus terminus bis accipitur; ut A : B :: C : D.

10. Si dentur plures termini (A, B, C, D &c.) continuè proportionales; Primus (A) habet ad Tertium (C) rationem *Duplicatam*, & ad Quartum (D) rationem *Triplicatam* ejus, quam idem Primus (A) habet ad Secundum (B.)

Sit *ex. gr.* 2: 4:: 4: 8, & 4: 8:: 8: 16 erit ratio primi 2 ad tertium 8 duplicata rationis primi 2 ad secundum 4. & ratio primi 2 ad quartum 16 triplicata rationis primi 2 ad secundum 4. Id est, quia secundus terminus 4 est duplum primi 2; tertius 8 erit duplum secundi 4, adeoque dupliciter duplum primi 2: & quartus 16, quia est duplum tertii 8, erit quadruplicitèr duplum primi 2. Et sic deinceps.

Similiter sit 1: 3:: 3: 9, & 3: 9:: 9: 27. Quia secundus 3 est triplum primi 1, tertius 9 est triplum secundi 3; adeoque tripliciter triplum primi 1: & quartus 27, quia triplum est tertii 9, est nonies triplum primi 1.

11. Termini sunt *Homologi*, qui ad sua Antecedentia vel Consequentia eandem habent rationem.

PROPOSITIO VII.

Si duæ quantitates (A, B) *fuerint æquales, eandem habebunt rationem ad tertiam quampiam* (Z) *& è converso.*

Hoc est, erit A: Z:: B: Z. & è converso Z: A:: Z: B.

PROPOSITIO VIII.

Si duæ [A & B] *fuerint inæquales; major* (A) *ad tertiam* [Z] *habebit majorem rationem, quàm minor* (B) *ad eandem* (Z).

Et è converso tertia quævis (Z) *minorem habet rationem ad quantitatem majorem* (A) *quàm ad minorem* (B.)

PROPOSITIO IX.

Si sit A: Z:: B: Z, *vel* Z: A:: Z: B; A & B *sunt æquales.*

PROPOSITIO X.

Si C *ad* Z *majorem rationem habet quàm* F *ad* Z, *erit* C *major*

major quàm F: *& si* Z *ad* F *majorem rationem habet quàm* Z *ad* C, *erit* F *minor quàm* C.

PROPOSITIO XI.

Rationes eidem tertiæ æquales, sunt æquales inter se.

Hoc est, si sit A: B:: X: Z, & C: D:: X: Z; erit A: B:: C: D.

PROPOSITIO XII.

Si A, B, C, *singulæ rationem eandem habuerint ad singulas* F, I, L; *erit ut* A *ad* F, *ita complexum omnium* A, B, C, *ad complexum omnium* F, I, L.

Id est, si singulæ A, B, C sint *v. g.* triplæ singularum F, I, L simul sumptæ A, B, C simul sumptæ triplæ simul sumptarum F, I, L.

PROPOSITIO XV.

Partes similes sunt inter se, ut sua tota.

Patet; nam illæ dicuntur partes *similes*, quæ eodem modo continentur in suis totis; id est, quæ sunt totorum dimidiæ, tertiæ, quartæ &c.

PROPOSITIO XVI.

Si sit A: B:: C: D; *erit* (*permutando, id est comparando primum cum tertio, & secundum cum quarto*) A: C:: B: D.

Sint enim B & D *v. g.* minores quàm A & C; erit (*per* 15. *h*) ratio totorum A & C, eadem quæ partium similium B & D.

Corollarium.

Si sit A: B:: C: D; *erit* (*invertendo, id est comparando secundum cum primo, & quartum cum tertio*) B: A:: D: C.

Patet; nam si A continet B, sicut C continet D; B continetur in A, sicut D in C.

PROPOSITIO XVII.

Si sit A: B:: C: D; *erit* (*dividendo*) A — B: B:: C — D: D.

Nam A & C mulctatæ similibus partibus, retinent (*ut patet*) eandem rationem ad se invicem: ergo (*per* 15. *h*) & ad suas partes similes B. & D.

PROPOSITIO XVIII.

Si sit A: B:: C: D; *erit* (*componendo*) A + B: B:: C + D: D.

Nam totæ A & C dum augentur partibus similibus [*per* 12. *h*] servant eandem proportionem inter se: ergo (*per* 15. *h*) & ad suas partes similes B & D.

PROPOSITIO XIX.

Si sit totum A *ad ablatum* B, *ut totum* C *ad ablatum* D; *erit* A: C:: A —B: C —D.

Nam si totis A & C auferantur partes similes B & D relinquentur partes similes A —B & C —D, quæ (*per* 15. *h*) sunt inter se, ut suæ totæ A & C.

Corollarium.

Si totum A *sit ad sui partem* B, *ut totum* C *ad sui partem* D; *erit* (*per conversionem rationis*) A: A —B:: C: C —D.

Nam [*per* 19. *h*] A: C:: A —B: C —D. Ergo (*permutando*) A: A —B:: C: C —D.

PROPOSITIO XXII.

Si sit A: B:: C: D, & B: E:: D: F; *erit* (*ex æquo ordinato*) A: E:: C: F.

Nam quia ponitur A: B:: C: D; erit (*permutando*) A: C:: B: D. Et quia ponitur B: E:: D: F; erit (*permutando*) B: D:: E: F. Ergo (*per* 11. *h*) A: C:: E: F; & (*permutando*) A: E:: C: F.

Corollarium.

Si sit A: B:: C: D, *&* E: F:: I: L; *erit* (*multiplicatis inter se utriusque Analogiæ terminis*) AE: BF:: CI: DL.

Hoc est rectangulum sub A & E erit ad rectang. sub B & F: ut rectang. sub C & I, ad rectang. sub D & L.

Nam (*per* 1. *l.* 6. quæ ab hac non dependet) BE: BF:: E: F, & DI: DL:: I: L. Sed [*ex suppos.*] E: F:: I: L. Ergo (*per* 11. *h*) BE: BF:: DI: DL.

Rursus [*per* 1. *l.* 6] AE: BE:: A: B, & CI: DI:: C: D. Sed (*ex suppos.*) A: B:: C: D. Ergo (*per* 11. *h*) AE: BE:: CI: DI.

Atqui (*ex dictis*) BE: BF:: DI: DL. Ergo (*ex æquo ordinato*) AE: BF:: CI: DL.

PROPOSITIO XXIII.

Si sit 1A: 2B:: 1D: 2F, *&* 2B: 3E:: 3C: 1D; *erit* (*ex æquo perturbato*) 1A: 3E:: 3C: 2F.

Fiat ut B ad E, sic F ad aliquam aliam O.

Jam quia est [*ex suppos.*] A: B:: D: F, & (*ex constr.*) B: E:: F: O; erit (*per* 22. *h*) A: E:: D: O.

Rursus quia sunt (*ex suppos.*) B: E:: C: D, & [*ex constr.*] B: E:: F: O; erit [*per* 11. *h*] C: D:: F: O, & (*permutando*) C: F:: D: O.

Sed est (*ex dictis*) A: E:: D: O. Ergo (*per* 11. *h*) A: E:: C: F.

PROPOSITIO XXIV.

Si sit A: B:: C: D, & E: B:: F: D; *erit* A ✣ E: B:: C ✣ F: D.

Quia est [*ex suppos.*] E: B:: F: D; erit (*invertendo*) B: E:: D: F. Cùm igitur sit (*ex suppos.*) A: B:: C: D, & (*ex dictis*) B: E:: D: F; erit (*per* 22. *h*) A: E:: C: F. Ergo (*componendo*) A ✣ E: E:: C ✣ F: F. Sed etiam [*ex suppos.*] E: B:: F: D. Ergo [*per* 22. *h*] A ✣ E: B:: C ✣ F: D.

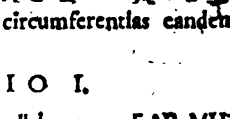
EX LIBRO SEXTO.

DEFINITIONES.

1. Figuræ dicuntur *similes*, quæ sibi mutuò sunt æquiangulæ, & latera æqualibus angulis opposita habent proportionalia.

2. Figuræ sunt *reciprocæ*, cùm in utraque antecedentes & consequentes rationum termini fuerint. Sic in parallelogrammis X, Z, (*Vide Fig. Prop.* 14.) Antecedens AC: Consequens OB:: Antecedens FO: Consequens CL.

3. *Altitudo* figuræ est longitudo perpendiculi DC ab ejus vertice ad basim ducti. Unde triangula æquè alta sunt inter easdem parallelas: & è contrà.

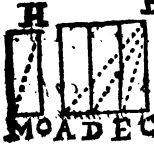


4. Arcus sunt *similes* qui ad suas circumferentias eandem habent rationem.

PROPOSITIO I.

Triangula (ABC, MHO) & *Parallelogramma* [AB, MH] *habentia eandem altitudinem, sunt inter se ut bases.*

1. *Pars.* Sit MO tertia pars baseos AC. Dividatur AC in tres partes æquales AD, DE, EC, & ducantur lineæ BD, BE.



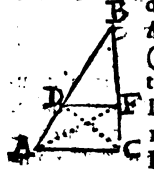
Quia triangula ABD, DBE, EBC habent [*ex suppos.*] eandem altitudinem, & (*ex constr.*) bases æquales, sunt (*per* 38. *l.* 1.) inter se æqualia; & (ob eandem rationem) eorum quodvis æquatur triangulo MHO: ergo MHO est tertia pars totius ABC. Sed (*ex suppos.*) basis MO est tertia pars baseos AC: ergo triangula ABC, MHO sunt inter se ut bases AC, MO.

2. *Pars.* Cùm Parallelogramma AB, MH sint (*per* 41. *l.* 1.) dupla triangulorum ABC, MHO; erunt etiam inter se ut bases.

PROPOSITIO II.

Si in triangulo (ABC) *ducatur aliqua* (DF) *parallela ad unum latus* (AC) *secabit alia latera* (AB, BC) *proportionaliter; id est, erit* BD: DA:: BF: FC.

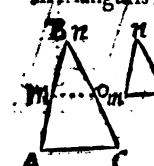
Ducantur AF, CD. Triangula DAF, DCF habentia eandem basim DF, & inter easdem parallelas DF, AC, sunt (*per* 37. *l.* 1.) inter se æqualia: ergo (*per* 7. *l.* 5.) eandem rationem habent ad triangulum DBF. Sed (*per* 1. *b*) DAF: BDF:: DA: BD. & DCF: DBF:: FC: BF. Ergo (*per* 11. *l.* 5.) DA: BD:: FC: BF. & (*invertendo*) BD: DA:: BF: FC.



PROPOSITIO IV.

Triangula sibi mutuò æquiangula sunt similia.

In triangulis ABC, MNO sint æquales anguli B & N, A & M, C & O. Si angulus N ponatur super æqualem B, latera NM, NO incident in latera BA, BC; & anguli NOM & BCA erunt (*ex suppos.*) æquales: ergo (*per* 29. *l.* 1.) AC & MO sunt parallelæ: ergo (*per* 2. *b*) AM: MN:: CO: ON. Et (*componendo*) AN seu AB: MN:: CN seu CB: ON. Et (*permutando*) AB: CB:: MN: NO.



Corollaria.

Si in triangulo aliquo (ABC) *ducatur* (MO) *parallela ad*

latus aliquod (AC;) *sient duo triangula similia* (BMO, BAC.)

Nam angulus B est communis: & externi BMO, BOM (*per* 27. *l.* 1.) æquantur internis BAC, BCA: ergo triangula sunt æquiangula, adeoque similia.

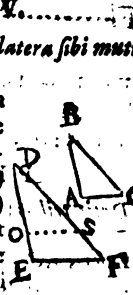
2. *Si duo triangula* (ABC, MNO) *habeant quosvis duos angulos* (A & M, B & N) *æquales sunt similia.*

Nam (*per* 5. *Corol. p.* 32. *l.* 1.) tertius angulus C æquatur tertio O: ergo &c.

PROPOSITIO V.

Si triangula (ABC, DEF) *habeant omnia latera sibi mutuò proportionalia, sunt sibi mutuò æquiangula.*

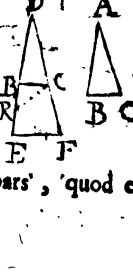
Sit *v. gr.* DE: BA:: EF: AC. & sint latera DE, DF majora quàm BA, BC. ex DE accipe DO æquale lateri BA, & per O duc OS parallelam lateri EF. erit (*per* 1. *Corol. p.* 4. *h*) ED: OD seu AB:: EF: OS. Sed (*ex suppos.*) ED: AB seu OD:: EF: AC. Ergo æqualia sunt OS & AC. Similiter ostendam DS & BC esse æqualia: ergo triangula ODS, ABC sunt sibi mutuò æquilatera; adeoque (*per* 8. *l.* 1.) etiam æquiangula. Sed ODS, EDF (*ut patet ex* 27. *l.* 1.) sunt æquiangula; ergo EDF, ABC sunt æquiangula.



PROPOSITIO VI.

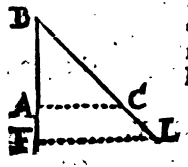
Si triangula (EDF, BAC) *habeant circa angulos æquales* (D & A) *latera* (ED, FD, AB, AC) *proportionalia, id est,* ED: AB:: FD: AC vel FD: AC:: ED: AB; *triangula sunt similia.*

Si angulus A ponatur super æqualem D; AB cadet in DE, & AC in DF. Jam si BC, EF sint parallelæ, triangula (*per* 1. *Corol. p.* 4. *h*) sunt similia. Si neges esse parallelas, ducatur CR Parallela ad EF: ergo (*per* 1. *Corol. p.* 4. *h*) FD: CD seu CA:: ED: RD. Sed (*ex suppos.*) FD: CA:: ED: BA seu BD. Ergo æqualia sunt RD, BD, totum & pars, quod est absurdum.



PROPOSITIO IX.

Rectam (BF) *secare secundum proportionem datam* (LC ad CB.)



Datis BF, BL constitutis ad angulum quemvis FBL, ducatur FL, eique parallela AC: erit (*per* 1. *Corol. p.* 4. *b*) FA: AB:: LC: CB.

PROPOSITIO XI.

Rectis (AB, BC) *invenire tertiam proportionalem.*

Ex AB productâ accipe BF æqualem datæ BC, & per A & F duc parallelas AC, FL. erit (*per* 1. *Corol. p.* 4. *b*) AB: BC:: BF seu BC: BL.

PROPOSITIO XII.

Datis tribus [BA, BC, BF] *invenire quartam proportionalem.*

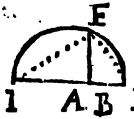
Ductis AC, FL parallelis, erit AB: BC:: BF: BL.

PROPOSITIO XIII.

Datis duabus rectis [IB, BF] *invenire mediam proportionalem.*

Totâ compositâ IF bisectâ in A, centro A, describe circulum per I & F; & ex B erige perpendicularem BE occurrentem circulo in E. erit IB: BE:: BE: BF.

Ducantur IE, EF. In triangulis IEF, IEB, recti sunt, adeoque æquales, angulus E [*per* 31. *l.* 3.] & angulus B (*per constr.*) & angulus I communis: ergo (*per* 5. *Corol. p.* 32. *l.* 1.) triangula IEF, IBE sunt æquiangula. Similiter in triangulis IEF, FBE, anguli E & B sunt recti, & F communis: ergo triangula IEF, FBE sunt æquiangula: ergo (*per ax.* 1.) triangula IBE, FBE sunt æquiangula, adeoque (*per* 4. *b*) similia; ergo IB: BE:: BE: BF.



Corollaria.

1. In triangulo rectangulo perpendicularis (EB) secat

triangulum (IEF) in partes (IEB, EFB) toti & inter se similes.

2. EB est media proportionalis inter IB, BF; & EF inter IF, BF; & IE inter IF, IB.

3. Hinc habetur alius modus facilis inveniendi mediam proportionalem inter duas datas, *v. g.* IF & IB Bisectâ majore IF, per I & F duc semicirculum; & IB posito super IF ex I in B, à puncto B excita perpendiculum BE ad peripheriam terminatum. Ducatur denique IE, & erit (ut ostensum est) IB: IE:: IE: IF.

PROPOSITIO XIV. & XV.

Æqualia Parallelogramma (AILC, FOBS) *& triangula* (FAC, OLB) *habentia unum angulum* (C) *uni* (O) *æqualem, habent latera circa æquales angulos reciproca; id est*, AC: OB:: FO: CL.

Et quæ latera habent reciproca, sunt æqualia.

XIV. 1. *Pars.* SB, IL productæ concurrant in Q, fintque tria parallelogramma X, Z, R. *Per* 1. *h.* X: R:: AC: OB; & Z: R:: FO: CL. Sed quia sunt (*ex suppos.*) æqualia X & Z, est (*per* 7. *l.* 5.) X: R:: Z: R; ergo (*per* 11. *l.* 5.) AC: OB:: FO: CL.

2. *Pars.* Est (*per* 1. *h*) X: R:: AC: OB; & Z: R:: FO: CL. Sed (*ex suppos.*) AC: OB:: FO: CL: ergo (*per* 11. *l.* 5.) X: R:: Z: R. Ergo (*per* 9. *l.* 5.) X & Z sunt æqualia.

XV. Ducatur AL, & similiter demonstres in triangulis FAC, OLB, esse AC: OB:: OL: FC.

PROPOSITIO XVI.

Si quatuor recta (AC, OB, FO, LC) *sint proportionalia, rectangulum* (X) *sub extremis* (AC, LC) *æquatur rectangulo* (Z) *sub mediis* (OB, FO.)

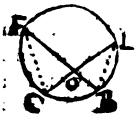
Patet (*ex.* 14. *h*;) nam habebunt latera reciproca; est enim (*ex suppos.*) AC: OB:: FO: LC.

Corollarium.

Si in circulo se secuerint duo recta (QL, BF) *rectangulum*

(COL:) *sub segmentis unius æquatur rectangulo* (BOP) *sub segmentis alterius.*

Ducantur CF, BL. Angulus F (*per* 21. *l.* 3.) æquatur angulo L insistenti eidem arcui CB, & anguli ad O (*per* 15. *l.* 1.) sunt æquales: ergo (*per* 2. *Corol. p.* 4. *h*) CO: OF:: BO: OL. Ergo (*per* 16. *h*) rectangulum sub CO, OL æquatur rectangulo sub OF, BO.



PROPOSITIO XVII.

Si tres recta (AB, FL BC) *sint proportionales, rectangulum sub extremis* (AB, BC) *æquatur quadrato mediæ* (FL.)

A B F L L O B C

Accipe LO æqualem FL, & erit (*per* 7. *l.* 5.) AB: FL:: LO: BC. Ergo [*per* 16. *h*] rectangulum sub AB, BC æquale est rectangulo sub FL, LO, id est [*per def.* 32. *l.* 1.] quadrato FL.

Corollaria.

Si ex puncto (D) *extra circulum posito ducatur tangens* (DB) *& utcunque secans* [DA] *erit quadratum tangentis* (DB) *æquale rectangulo* [ADC] *sub tota secante* (DA) *& parte* [DC] *extra circulum.*

Ducantur BC, BA. In triangulis ADB, BCD, angulus D communis est, & angulus DBC (*per* 32. *l.* 3.) æqualis angulo BAD in segmento alterno: ergo (*per* 2. *Corol. p.* 4. *h*) DA: DB: DB: DC: ergo (*per* 17. *h*) rectangulum ADC æquatur quadrato DB.

2. *Duæ rectæ quæ ex eodem puncto* [D] *circulum tangunt, sunt æquales.*

Nam earum quadrata æquantur singula eidem rectangulo ADC.

3. *Quotcunque circulum secent ex eodem puncto* [D] *omnia rectangula erunt æqualia.*

Nam singula æquantur quadrato DB.

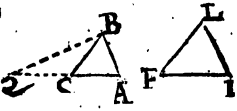
PROPOSITIO XIX.

Triangulorum similium [ABC, FLI] *proportio est duplicata proportionis laterum* (AC, FI) *æqualibus angulis* (B & L) *subtensorum.*

Hoc

Hoc est si fiat ut AC ad FI, sic FI ad AQ; erit ABC: FLI:: AC: AQ.

Quia triangula ponuntur similia, est (*per 4. h*) AB: IL:: AC: IF. Id est (*per constr.*) ut IF ad AQ. Ergo in triangulis QBA, FLI latera circa æquales angulos A & I sunt reciproca. Ergo (*per 15. h*) QBA, FLI sunt æqualia: sed (*per 1. h*) ABC: QBA:: AC: AQ. Ergo (*per 7. l. 5.*) ABC: FLI:: AC: AQ.

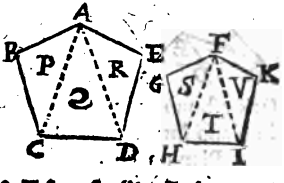


PROPOSITIO XXI.

PARS PRIMA.

Polygona similia dividuntur in similia triangula.

Ducantur rectæ AC, AD, FH, FI. In triangulis P, S, anguli B & G (*per 1. def. h*) sunt æquales, & BA: GF:: BC: GH. Ergo (*per 6. h*) P & S sunt similia: ergo (*per 1. def. h*) anguli BCA, GHF sunt æquales: ergo his subductis ex angulis BCD, GHI (*per 1. def. h*) æqualibus, reliqui anguli ACD, FHI æquales. Dein (*per 4. h*) AC: FH:: BC: GH. Sed (*ex suppos.*) BC: GH:: CD: HI. Ergo (*per 11. l. 5.*) AC: FH:: CD: HI. Ergo (*per 6. h*) triangula Q & T sunt similia. Eodem modo ostendas R & V, & quotquot fuerint, esse similia.



PARS SECUNDA.

Polygona similia sunt ad se invicem in proportione duplicata laterum (AB, FG) inter æquales angulos existentium.

Quia triangula P, S, & Q, T, & R, V, sunt similia, Ratio P ad S (*per 19. h.*) est duplicata rationis AB ad FG; & Ratio Q ad T duplicata rationis CD ad HI; & Ratio R ad V duplicata rationis DE ad IK: ergo (*per 12. l. 5.*) Ratio omnium P, Q, R, ad omnia S, T, V, id est Polygoni ad Polygonum, est duplicata rationis laterum AB, CD, DE ad FG, HI, IK, id est (quia hæc latera supponuntur homologa) duplicata rationis AB ad FG.

PROPOSITIO XXIII.

In circulis æqualibus anguli, sive ad centrum, sive ad peripheriam sunt inter se ut arcus subtensi.

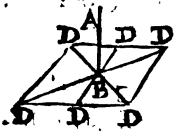
De angulis ad centrum demonstratur eodem modo quo prima hujus; tantùm loco Prop. 38. l. 1. citetur 29. l. 3.

Quoniam verò anguli ad centrum dupli sunt angulorum ad peripheriam; quod de illis ostenditur, de his etiam verum est.

EX LIBRO UNDECIMO.

Ex Libro hoc delegi, quæ ad Trigonometriam Sphæricam sunt maximè necessaria; adjecique nonnulla, quæ videbantur in Euclide desiderari.

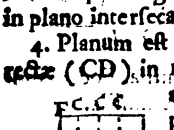
DEFINITIONES.

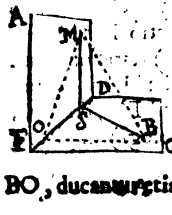


3. Linea (AB) dicitur *Recta* seu *Perpendicularis plano* (DD) si cum omnibus lineis (BD) in plano ductis, à quibus tangitur, efficiat angulos rectos (ABD, ABD.)

Ut linea aliqua sciatur esse plano recta, satis est quòd cognoscatur esse perpendicularis duabus rectis se in plano intersecantibus, ut perspicuum fiet *Prop. 4. h.*

4. Planum est *Rectum* seu *Perpendiculare plano*, si omnes rectæ (CD) in uno plano (FB) ductæ perpendiculariter ad sectionem communem (AB) sint alteri plano (EB) perpendiculares. Ex hac definitione sequitur, quòd quandocunque planum plano insistit perpendiculariter, non possit in eorum uno duci linea perpendicularis ad planorum sectionem communem, quin recta sit alteri plano:



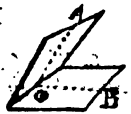


Imò nec duci potest in uno plano aliqua (MS) perpendicularis linea (BS) ductæ in altero plano obliquè ad sectionem communem (FD), quin illa (MS) sit alteri plano (CD) perpendicularis.

Ducatur BO perpendiculariter ad sectionem communem; factâque MS æquali ipsi BO, ducantur etiam MO, MB.

Nunc in triangulis MSB, MOB, angulus MSB est (*ex suppos.*) rectus, sicut & (*per hanc & praced. def.*) angulus MOB; latus MB commune, & latera MS, BO (*ex constr.*) æqualia. Ergo (*per 4. Corol. p. 47. l. 1.*) tertium latus MO æquatur tertio BS. Ergo in triangulis MOS, BOS habentur MS & BO, item MO & BS æqualia, & tertium (seu distantia OS) commune: ergo (*per 8. l. 1.*) angulus MSO æquatur angulo BOS (*ex constr.*) recto: ergo MS est perpendicularis sectioni communi FD, adeoque & plano CD.

6. *Plani ad planum inclinatio* est angulus acutus (AOB,) quem efficiunt rectæ (AO, BO) in utroque plano ductæ perpendiculariter ad sectionem planorum communem.



PROPOSITIO I.

Non potest linea recta una pars esse in plano, altera extra idem planum.

Patet ex defin. lineæ rectæ & plani; nam utrumque ducitur per viam brevissimam, quæ unica est inter eosdem terminos.

PROPOSITIO II.

Omne triangulum in uno est plano, & duæ rectæ se mutuò secantes sunt in eodem plano.

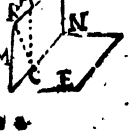
1. *Pars.* Si trianguli CBE aliqua pars FEB non sit in eodem plano, quo altera CEF; rectæ CB una pars CF erit in plano CFE, alia FB extra idem planum. Sed hoc est contra 1. *h.* Ergo &c.

2. *Pars.* Quia triangulum CBE est in uno plano, rectæ CE, BE, adeoque (*per 1. h*) totæ CD, BA sunt in eodem plano.

Corollarium.

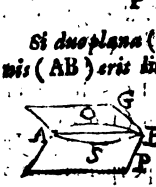
Ex eodem puncto (C) *non possunt erigi duæ* (CA, CR) *perpendiculares ad idem planum* [EN.]

Nam ducto per illas plano MN, anguli RCN, ACN (*per 3. def. h*) erunt recti, adeoque æquales, quod est absurdum.



PROPOSITIO III.

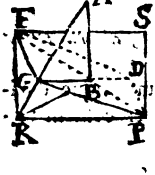
Si duo plana (AG, AP) *se mutuò secuerint, sectio communis* (AB) *erit linea recta.*



Si neges; à puncto A ad B duc in plano AG rectam AOB, & in plano AP rectam ASB; sic duæ rectæ AOB, ASB spatium includent; quod est absurdum.

PROPOSITIO IV.

Si linea (AB) *sit recta duabus* (FP, GD) *se mutuò secantibus, est recta plano* (FGPD) *per ipsas ducto.*

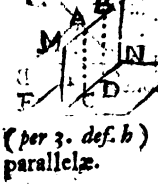


Si neges; per B transeat planum RS, cui AB sit recta; & per lineas AB, BG ducatur aliud planum triangulare ARB secans planum RS alicubi in RB. Erit angulus ABR (*per 3. def. h*) rectus, adeoque æqualis angulo ABG (*ex suppos.*) recto; quod est absurdum.

PROPOSITIO VI.

Si linea (AC, BD) *sint eidem plano rectæ, sunt parallelæ.*

Super CD erigatur planum MN rectum plano EF. Jam si AC, BD non essent in plano MN, possent ex C & D erigi in



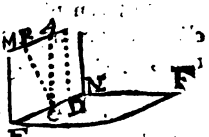
plano MN aliæ perpendiculares plano EF; sicque ab eodem puncto possent excitari duæ perpendiculares eidem plano; quod est contra *Corol. p. 2. h.* Ergo sunt in plano MN. Cùm igitur anguli ACD, BDC sint (*per 3. def. h*) recti, lineæ AC, BD (*per 29. l. 1.*) sunt parallelæ.

PROPOSITIO VIII.

Si Parallelarum (AC, BD) *una* (BD) *sit recta plano* EF) *altera* (AC) *erit eidem recta.*

Super CD erigatur planum MN rectum plano EF. Quod BD, cùm sit (*ex suppos.*) recta plano EF, sit in plano MN,

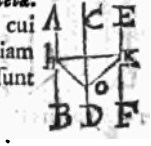
ostenditur ut in *Prop. 6. h.* Ergo cùm AC, BD ponantur parallelæ, etiam AC (*per 36. def. l. 1.*) erit in plano MN. Jam si AC non sit recta plano EF, ex C erigatur CR in plano MN recta plano EF. Erunt (*per 6. h*) CR, DB parallelæ: ergo (*per 30. l. 1.*) CR est etiam parallela ad AC: ergo duæ parallelæ CR, AC concurrunt in C; quod est absurdum.



PROPOSITIO IX.

Rectæ (AB, EF) *si sint eidem* (CD) *parallelæ, licèt non sint omnes in eodem plano, sunt inter se parallelæ.*

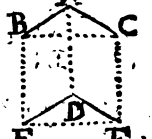
Rectas AB, EF, CD secet planum OKH, cui perpendicularis sit CD: erunt (*per 8. h*) etiam AB, EF eidem plano rectæ: ergo (*per 6. h*) sunt inter se parallelæ.



PROPOSITIO X.

Si duæ rectæ (AB, AC) *sint duabus* (DE, DF) *parallelæ, etsi in planis diversis, faciunt angulos* (A & D) *æquales.*

Fiant æquales AB & DE, AC & DF; & ducantur AD, CF, BE, CB, FE. Quia AB, DE sunt æquales (*ex constr.*) & (*ex suppos.*) parallelæ, AD, BE sunt (*per 33. l. 1.*) æquales & parallelæ: item quia AC, DF sunt parallelæ & æquales, AD, CF sunt (*per 33. l. 1.*) parallelæ & æquales: ergo BE, CF sunt (*per ax. 1.*) æquales, & (*per 9. h*) parallelæ: ergo (*per 33. l. 1.*) CB, FE sunt parallelæ & æquales: ergo triangula BAC, EDF sunt sibi mutuò æquilatera: ergo (*per 8. l. 1.*) anguli BAC, EDF sunt æquales.



PROPOSITIO XLII.

Linea ex eodem puncto ducta nequeunt ambæ ad idem planum esse rectæ.

Vide *Corol. p. 2. h.*

PROPOSITIO XIV.

Si eadem recta (AG. Vide Fig. Prop. 15.) *sit duobus planis* (BC, EF) *perpendicularis, plana sunt parallela.*

Paret; nam planum unum BC non posset ad aliud EF inclinari, quin inclinetur ad rectam AG inter plana intercepta, quæ proinde non foret perpendicularis.

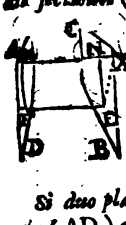
PROPOSITIO XV.

Si duæ rectæ (BA, CA) *se mutuò tangentes, ad alias duas* (ED, FD) *se mutuò tangentes, sint parallelæ; plana per ipsas ductis erunt parallela.*

Ex A ducatur AG, recta plano EF, & ponantur GH, GI parallelæ ad DE, DF. Erunt (*per 9. h*) GH, GI etiam parallelæ ad AB, AC. Cùm igitur anguli IGA, HGA (*per 3. def. h*) sint recti, erunt (*per 27. l. 1.*) anguli CAG, BAG recti: ergo (*per 4. h*) AG est recta plano BC: sed (*ex constr.*) est etiam recta plano EF: ergo (*per 14. h*) plana BC, EF sunt parallela.

PROPOSITIO XVI.

Si planum (FONE) *secet plana parallela* (AD, CB,) *faciat sectiones* (FO, EN) *parallelas.*



Nam si non sint parallelæ, cùm sint in eodem plano FONE, productæ alicubi concurrent, adeoque concurrent etiam plana parallela AD, CB, in quibus istæ lineæ FO, EN existunt; quod est absurdum.

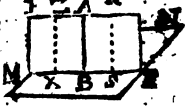
Corollarium.

Si duo plana (AD, CB) *fuerint parallela, & in eorum uno* (AD) *ducatur recta* (FO) *& huic* (FO) *altera parallela* (EN,) *cujus extremum* (E) *sit in alio plano* (CB;) *erit tota* (EN) *in hoc plano* (CB.)

Per rectas FO, EN, ducatur planum FONE, & producatur donec faciat planum CB. Erit EN sectio planorum CB, FONE; adeoque parallela ad FO. Si neges; sit EX illa sectio: ergo (*per 16. h*) EX, FO sunt parallelæ. Sed (*ex supp.*) FO, EN sunt parallelæ: ergo (*per 30. l. 1.*) EX, EN, sunt parallelæ: quod est absurdum, cùm concurrant in E.

PROPOSITIO XVIII.

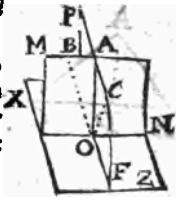
Si recta (AB) *sit perpendicularis plano* (MN) *omnia plana per ipsam* (AB) *ducta sunt isti plano* (MN) *perpendicularia.*



Per AB transeat planum FP, in quo ducantur quotcunque ZX, RS parallelæ ad AB. Erunt ZX, RS (*per 8. h*) rectæ plano MN: ergo (*per 4. def. h*) planum FP est eidem rectum.

PROPOSITIO XIX.

Si plana (MN, PF) *se mutuò secantia sint recta plano* (XZ) *eorum sectio communis* (OA) *est eidem* (XZ) *perpendicularis.*



Si neges; ducantur ex O in plano MN recta OB, & in plano PF recta OC, perpendiculares ad XZ. Erunt (*per 6. h*) OB, OC parallelæ; quod est absurdum, cùm se tangant in O.

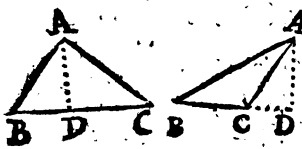
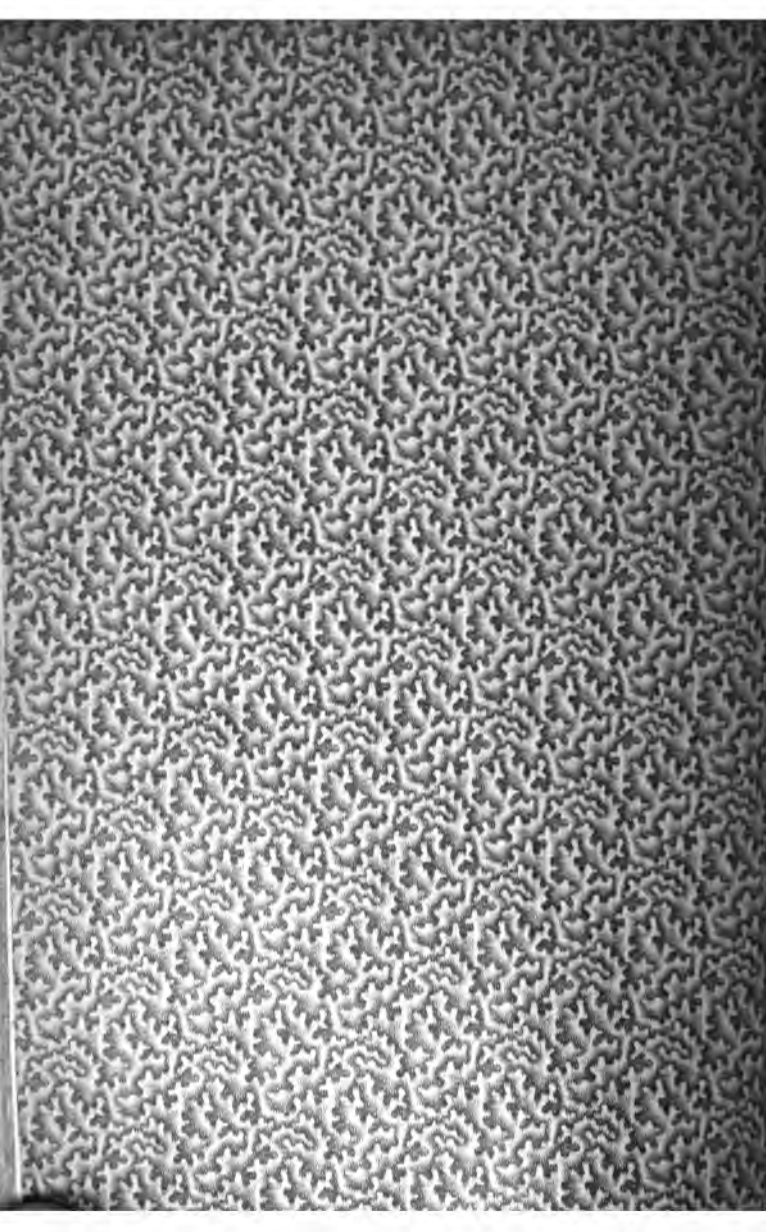
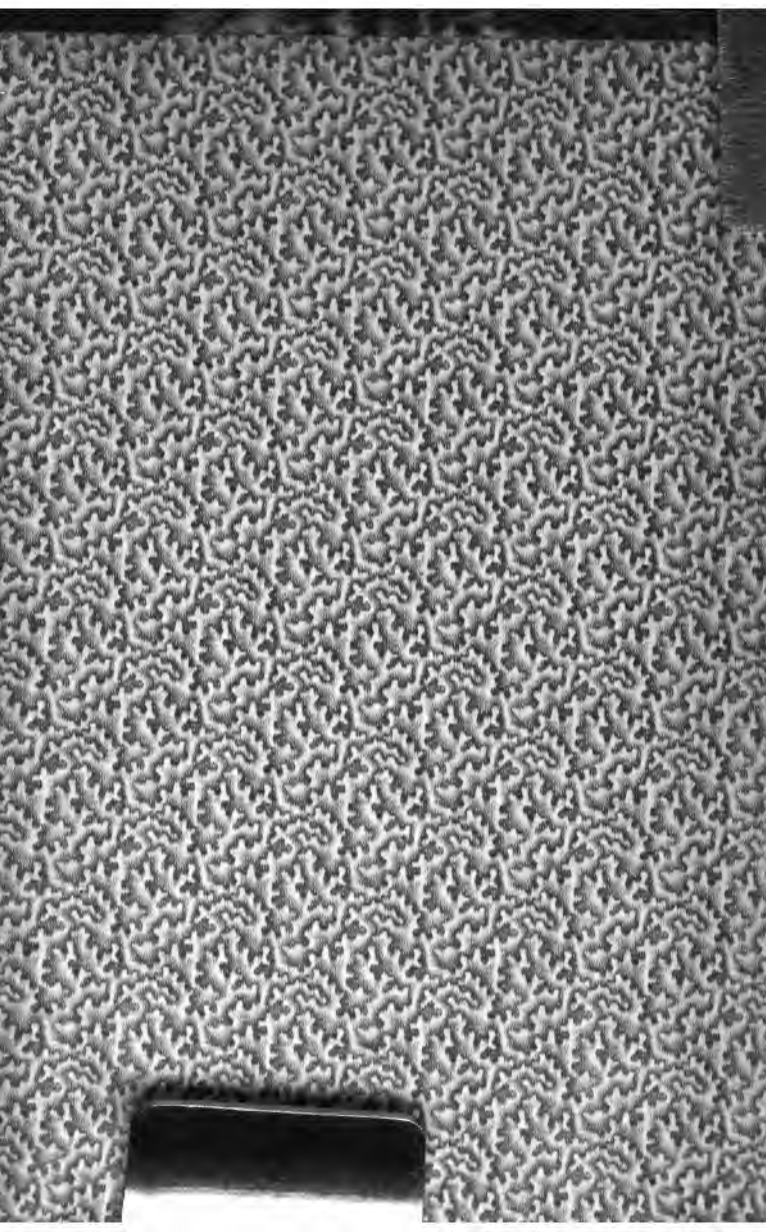
Corollarium.

Si per duo plana (AB, AC) *ad se mutuò inclinata ducatur aliud* (BOS) *eorum uni* (AC) *rectum, secabit utrumque* (AB, AC) *vel neutrum, perpendiculariter ad eorum sectionem communem* (AM.)

Si neges; secet planum AB in BS perpendiculariter, & AC in OS obliquè ad sectionem communem AM. Ex S erigatur SD in plano AC perpendicularis ad AM. Jam quia AM est ad BS (*ex suppos.*) & ad SD (*ex constr.*) recta; est (*per 4. h*) recta plano BSD: ergo planum AC (*per 18. h*) est rectum plano BSD. Sed idem AC est [*ex suppos.*] rectum plano BOS: ergo planorum BSD, BOS sectio communis BS est (*per 19. h*) recta plano AC: ergo (*per 18. h*) planum AB est rectum plano AC, ad quod supponitur inclinarum; quod est absurdum. Ergo &c.

FINIS.

Pag.	Lin.	Errata.	Corrige.
12.	34. & 35.	AMC.	AMB.
28.	33.	10,	10, 0:
29.	18.	*l.* 1.	*l.* 6.
34.	1.	& his	Ex his.
42.	2.	*Prop.* 9. 15.	*Omnia.*
44.	22.	concurrant.	concurrent.
52.	23.	(*per* 23. *l.* 1.)	(*per* 29. *l.* 3.)
63.	3.	(*per* 1, *c.* 1. *h*)	(*per* 1. *c.* 2. *h*)
69.	37.	(*per* 2. *Cor.*)	(*per* 3. *Cor.*)
132.	2.	BC in.	BC utcunque in
Ibid.	8.	EF.	BC
134.	3.	per E.	per D & E
150.	2.	(*per* 17. *b*)	(*per* 16. *b*)
151.	26.	quantitatem.	quantitate,
Ibid.	31.	A ≅	A = B.



Prætereà hæ Figuræ apponi debuerant pag. 15. *Notando.* 2. cui Figuræ ibidem positæ non satis congruunt.